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BENDING OF BEAMS

Textbook
educational and methodological manual for the course «Resistance of materials»

for students of mechanical engineering specialties

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The manual contains calculations of straight beams with direct transverse bending. Intended for students of mechanical engineering majors. Basic theoretical information, examples of solving typical problems, as well as variants of tasks for independent work are presented.

The manual be useful for teachers, as well as for postgraduate students and researchers.

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Introduction

The current stage of scientific and technological development requires the improvement of strength calculation methods in order to introduce new technologies, increase the reliability and durability of machines, as the competitiveness of engineering products on the world market is impossible without a sharp improvement in quality machines.

The textbook is one of a series of educational and methodological literature prepared at the Department of Resistance of Materials of NTU «KhPI» in order to fill the gap created in recent years in the publication of educational literature, in particular, the course «Resistance of materials and calculations for strength in mechanical engineering».

The manual covers one of the important sections of the general course of resistance of materials, namely, calculations of bending, and is intended for students to master the general provisions of the theory of flat direct bending of beams, acquaintance with the examples.

The first section of the manual considers the bending of rectilinear rods, the definition of internal force factors in direct transverse bending. The second section considers the definition of normal stresses in pure bending, tangential stresses in transverse bending of beams, calculations for strength taking into account normal and tangential stresses.

The third section considers the definition of displacements in direct bending using the differential equation of the elastic line and the Mohr integral. The fourth section presents the concept of geometric characteristics of the cross-sections of the rods. The fifth section provides calculation schemes and numerical data for individual calculation and design tasks, as well as examples of their solution and design.

1. BENDING OF STRAIGHT BEAMS

1.1. Classification of bending and types of supports

Bending is a type of rod deformation in which bending moments occur in its cross sections.

Classification of bending. Bending is divided into transverse - when the external load acts in the direction perpendicular to the axis of the rod, longitudinal - when external forces act along the axis of the rod and longitudinal - transverse.

Transverse bending is divided into flat, in which the bending forces lie in one plane, and spatial, in which the external bending forces are arbitrarily oriented in space.

Flat bending is divided into straight and oblique. In the case of direct bending, the plane of action of bending loads coincides with one of the main axes of inertia of the section.

Figure 1.1 shows the case of loading the rod during direct transverse bending. External forces are located in the plane YOZ, which coincides with the main axis of section Y. In oblique bending, the plane of action of bending loads does not coincide with any of the main axes of inertia.

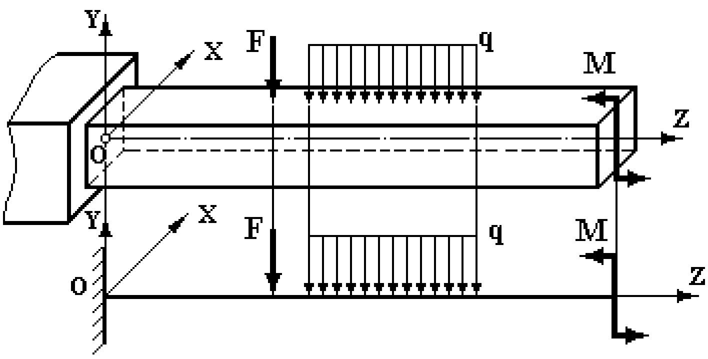


Figure 1.1 - Direct transverse bending

A special case of transverse bending ($Q_y \neq 0$ and $M_x \neq 0$) is pure bending, in which the shear force Q_y is zero, and the bending moment M_x is the only internal force factor in the cross section of the rod and is constant in the rod.

Consider the bending of beams. A beam is a rod that is attached to the supports and works on bending.

The number of external connections in the supports prohibits the movement of the beam as a solid whole. Flat supports of beams and reactive forces in them are shown in Fig.1.2.

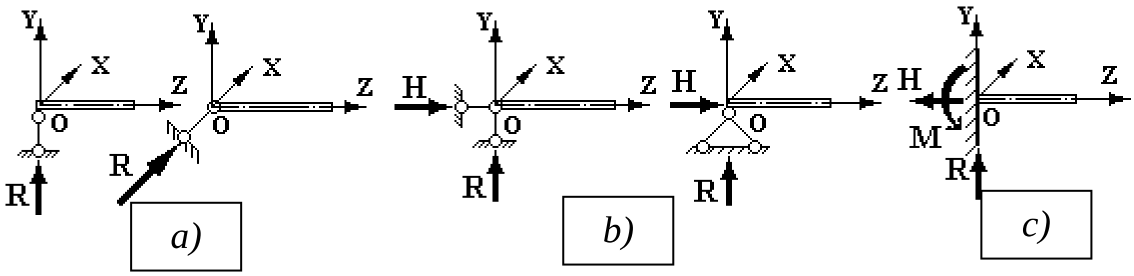


Figure 1.2 - Flat supports of beams

In the hinged support (Fig. 1.2, *a*) there is one reactive force R acting perpendicular to the support surface (in the direction of the shown connection). In the articulated-fixed support (Fig. 1.2, *b*) there are two components of the reaction: vertical R and horizontal H. In the clamp (rigid clamp) (Fig. 1.2, *c*), there are three components: vertical R, horizontal H and moment M.

For the kinematic immutability of flat beams, the required number of external connections is three, and in the case of flat bending, the horizontal component H of the reaction in the hinged-fixed support is identically equal to zero. Therefore, we further use two equilibrium equations. If the number of external transverse connections is more than two, then such a beam is called statically indeterminate (multi-support). Types and names of beams that are found are shown in Fig. 1.3.

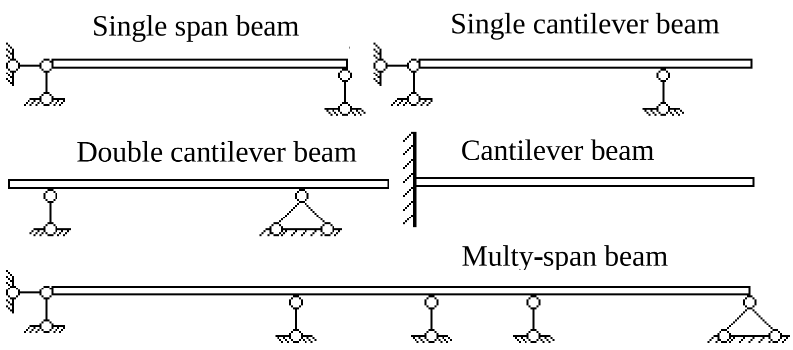


Figure 1.3 - Types and names of beams

1.2. Shear force Q_y and bending moment M_x as internal force factors during bending

Let's analyze the internal force factors in the cross section of the beam in direct transverse bending, and then formulate the basic rules for plotting diagrams Q_y and M_x.

Consider a cantilever rod with a clamped right end and loaded with forces F1 and F2 (Fig. 1.4). Let F1> F2.

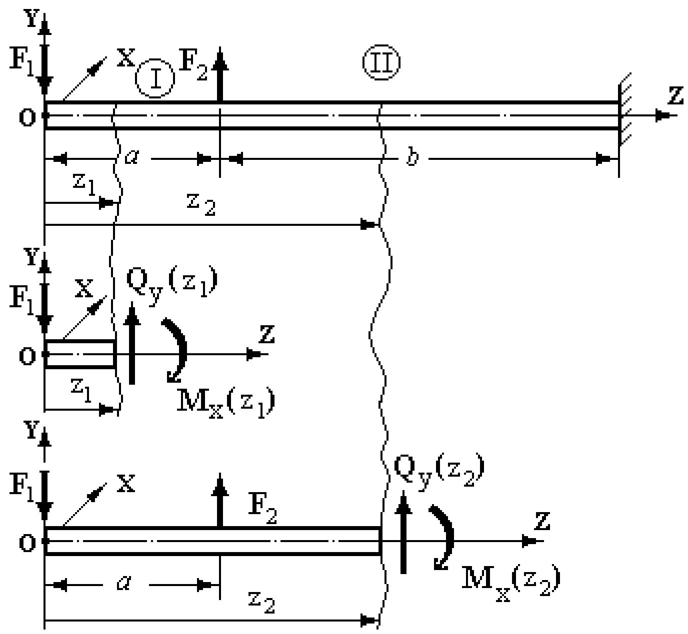


Figure 1.4 - Cantilever rod

Let's use the method of sections. We will choose a section on the first and second sites, we will show the cut off parts, we will replace action of the rejected parts on the left by internal force factors Q_y and M_x. From the conditions of statics (the sum of projections of forces on the Y axis and the sum of moments relative to the X axis passing through the center of gravity of the considered section) we determine their values.

<u>1 segment:</u> $\Sigma Fi = - F1 + Q_y(z1) = 0$, from where $Q_y(z1) = F1$;

$\Sigma M_i = F_1 \cdot z_1 - M_x(z_1) = 0$, from where $M_x(z_1) = F_1 \cdot z_1$.

<u>2 segment</u>, $\Sigma F_i = - F_1 + F_2 + Q_y(z_2) = 0$, from where $Q_y(z_2) = F_1 - F_2$;

$\Sigma M_i = F_1 \cdot z_2 - F_2 \cdot (z_2) - M_x(z_2 - a) = 0$, from where $M_x(z_2) = F_1 \cdot z_2 - F_2 \cdot (z_2 - a)$.

Using the following notation, we formulate the following rules for determining the transverse force and bending moment during bending.

Shearing Forces in the section – Q_y***(z) is numerically equal to the algebraic sum of projections on the normal (Y axis) to the axis of the rod of all forces located on one side of the section (all one-sided forces), and forms a substitution of the action of the rejected part on the left.***

<u>Rule of signs.</u> Shearing force is considered positive (positive) if it rotates the cut-off part of the beam relative to the center of gravity of the cross-section clockwise, and negative (negative) if it rotates counterclockwise.

Bending moments in the section – M_x***(z) is numerically equal to the algebraic sum of moments relative to the center of gravity of the cross section of all forces located on one side of the cross section (all one-sided forces), and forms a replacement for the action of the rejected part on the left.***

<u>Rule of signs.</u> The bending moment is considered positive (positive) if the cut part bends convexly downwards (compressed fiber at the top, stretched at the bottom), and negative (negative) - if vice versa. Thus, the plot of bending moments is built from the compressed fiber.

Schematically accepted rules of signs look like this:

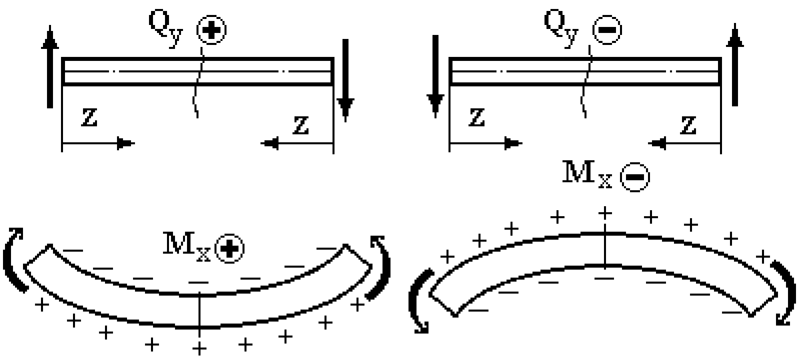


Figure 1.5 - Rules of signs

1.3. Differential dependences of bending

Consider a beam loaded with an arbitrary distributed load *q(z)* (Fig. 1.6, *a*) In the cross section at a distance z we select an element of length dz (Fig. 1.6, *a*).

In section I there are internal force factors Q_y and M_x, in section II at a distance dz from the first internal forces $Q_y + dQ_y$ and $M_x + dM_x$ act.

Within infinitesimal *dz*, the load *q(z)* can be considered uniformly distributed and equal to *q*.

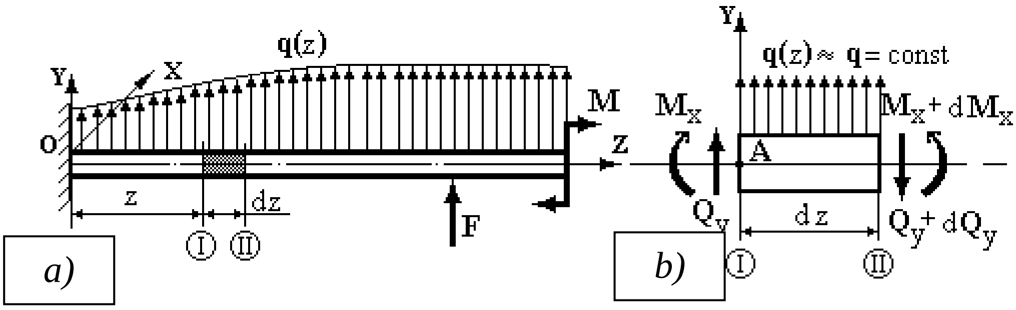


Figure 1.6 – Beam and its element *dz*

Since the beam under the action of external load is in equilibrium, then each of its elements under the action of external and internal forces is also in equilibrium (Fig. 1.6, *b*).

Let's write down the conditions of statics:

1. $\sum F_y = 0;\ Q_y - Q_y - dQ_y + qdz = 0$, from where $qdz - dQ_y = 0$, so

$$q = \frac{dQ_Y}{dz} \tag{1.1}$$

2. $\sum M_A = 0$; $-M_x + q \cdot dz \cdot \frac{dz}{2} - (Q_y + dQ_y) \cdot dz + M_x + dM_x = 0$, giving similar terms and despising infinitesimal second-order in comparison with infinitesimal first-order, we obtain: $-Q_y dz + dM_x = 0$, from where :

$$Q_y = \frac{dM_x}{dz}. \tag{1.2}$$

3. Substituting expression (1.2) into the dependence (1.1), we obtain:

$$q = \frac{dQ_y}{dz} = \frac{d^2 M_x}{dz^2}. \tag{1.3}$$

Differential dependences (1.2) and (1.3) allow us to establish some features of the distributions of shearing forces and bending moments.

The following rules can be used to build and test diagrams M_x and Q_y .

1. On segments where the distributed load is absent (q=0), the diagram Q_y is constant, and the diagram M_x represents a linear function.

2. On segments with evenly distributed load q, the diagram Q_y is linear, and the diagram M_x is a square parabola, and the convexity of the parabola is directed in the opposite direction of the distributed load. At the point $z = z^*$ where the transverse force $Q_y(z^*) = 0$ (changes the sign), the moment M_x reaches an extreme value ($M_{x\max}, M_{x\min}$).

3. In areas where $Q_y = 0$ the plot M_x is permanent.

4. The following points are formulated for the right z axis (for the right coordinate system). In the area where the shearing force Q_y is positive, the moment diagram M_x increases and decreases - if Q_y negative.

5. In sections where external concentrated forces are applied to the beam:

a) on the plot Q_y there are jumps in their magnitude and in the direction of the applied concentrated forces;

b) fractures appear on the plot M_x, and the edges of the fractures are directed against the action of concentrated forces.

6. In sections where concentrated moments are applied to the beam, jumps on the magnitudes of these moments are observed on the M_x plot.

7. The diagram Q_y is a diagram of the first derivative of the moment M_x function, ie the ordinates Q_y are proportional to the tangent of the angle of inclination tangent to the diagram M_x.

Next we will consider examples of construction of diagrams of shearing forces Q_y and bending moments M_x.

Example 1.

We show the current section with the coordinate z (Fig. 1.7), the limits of its change, write the functions Q_y and M_x. When taking into account the evenly distributed load q, we use the following method: replace it with the concentrated force $q \cdot z$ applied in the middle of the section (shoulder of concentrated force 0,5·z).

$0 \le z \le \ell \Rightarrow \quad Q_y(z) = q \cdot z \,;\; M_x(z) = -q \cdot \frac{z^2}{2}$. Next, calculate the value Q_y and M_x :

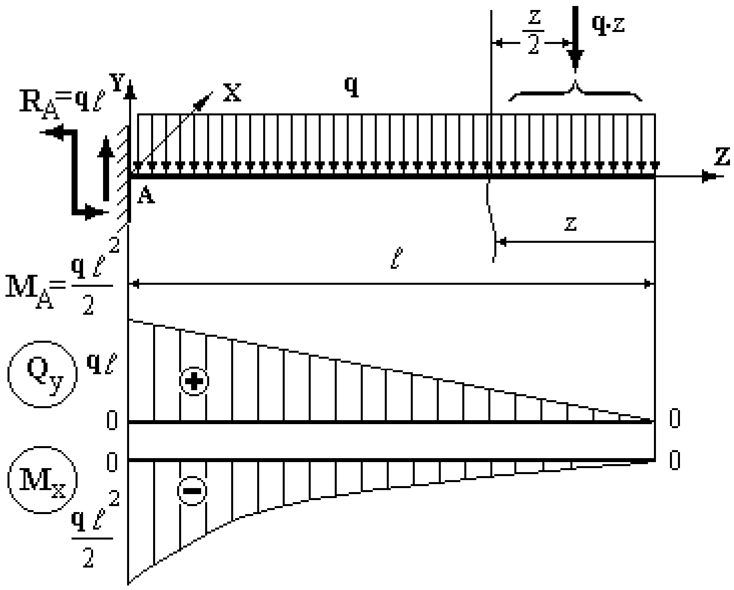


Figure 1.7 - Cantilever beam with evenly distributed load q

$$z=0 \Rightarrow Q_y=0 \quad ; \quad M_x=0\,; \qquad z=\ell \Rightarrow Q_y=q\cdot\ell\,; \quad \mathrm{M}_x=-\frac{q\cdot\ell^2}{2}\,.$$

According to the diagram, using the rules of verification, determine the reference reactions R_A and M_A. The reaction $R_A=q\cdot\ell$ is equal to the magnitude of the jump on the plot Q_y in this section and is directed upwards because Q_y is positive. If you build a diagram Q_y, going to the left, the reaction R_A should give a positive value Q_y, ie should be directed upwards. From the conditions of statics $\sum F_{yi}=R_A-q\cdot\ell=0$ we obtain the same value $R_A=q\cdot\ell$.

On the plot M_x in the jamming of the momentum jumps by magnitude $\frac{q\cdot\ell^2}{2}$, therefore $M_A=\frac{q\cdot\ell^2}{2}$. Due to the fact that M_x in the clamp is negative, M_A must be directed counterclockwise. From the condition of statics

$\sum M_{Ai}=-q\cdot\ell\cdot\frac{\ell}{2}+M_A=0$ we obtain:. $M_A=\frac{q\cdot\ell^2}{2}$.

Example 2.

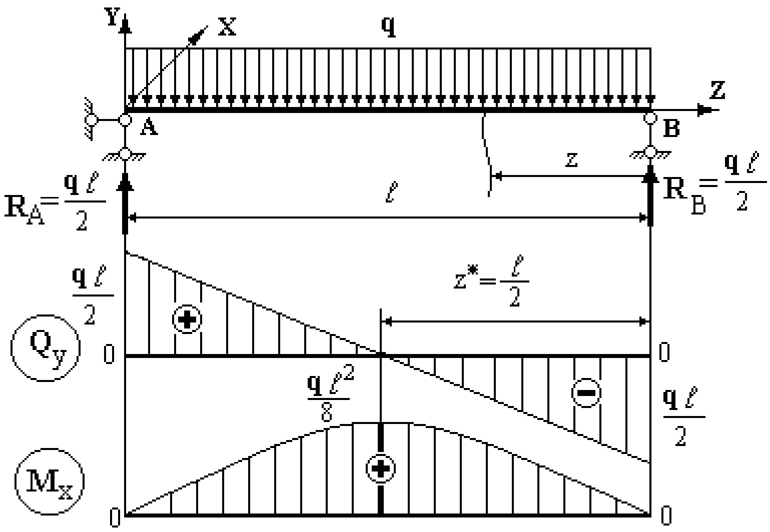


Figure 1.8 - Single span beam with evenly distributed load q

1. Let's define basic reactions.

$$\sum M_A = 0, \quad R_B \cdot \ell - q \cdot \ell \cdot \frac{\ell}{2} = 0, \text{ from where: } R_B = \frac{q \cdot \ell}{2}.$$

$$\sum M_B = 0, \ -R_A \cdot \ell + q \cdot \ell \cdot \frac{\ell}{2} = 0, \text{ from where } R_A = \frac{q \cdot \ell}{2}.$$

Verification: $\sum F_{yi} \equiv 0$, $R_A + R_B - q \cdot \ell \equiv \frac{q \cdot \ell}{2} + \frac{q \cdot \ell}{2} - q \cdot \ell \equiv 0$.

The scheme of the problem is symmetric, so both reactions are equal to half the external load.

2. We show the current section with the coordinate, the boundaries of its change and write down the functions Q_y and M_x:

$0 \le z \le \ell \Rightarrow$

$$Q_y(z) = q \cdot z - R_B = q \cdot z - \frac{q \cdot \ell}{2}; \ M_x(z) = -q \cdot \frac{z^2}{2} + R_B \cdot z = -q \cdot \frac{z^2}{2} + q \cdot \frac{\ell}{2} \cdot z.$$

Next, calculate the value Q_y and M_x :

$z = 0 \Rightarrow Q_y = -\frac{q\ell}{2}, M_x = 0$, $z = \ell \Rightarrow Q_y = +\frac{q\ell}{2}, M_x = 0$.

Note that at the point where $Q_y = qz^* - \frac{q\ell}{2} = 0$, the bending moment M_x must be of extreme magnitude. So, $z^* = \frac{\ell}{2}$ and

$$M_{x\max} = -\frac{qz^{*2}}{2} + \frac{q\ell}{2} z^* = -\frac{q\ell^2}{8} + \frac{q\ell^2}{4} = \frac{q\ell^2}{8}.$$

Example 3.

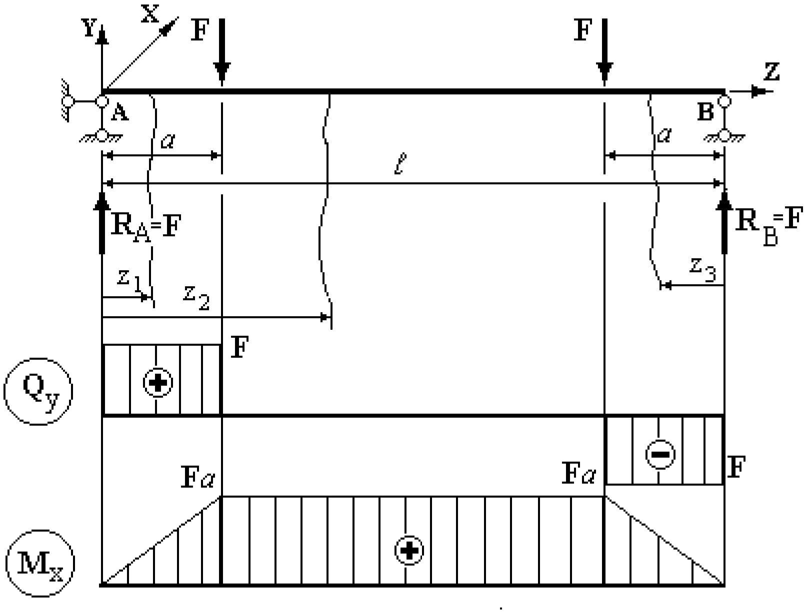


Figure 1.9 - Singlr span beam with two forces

1. Due to the symmetry of the problem $R_A = R_B = F$.
2. Write down the functions and determine the characteristic values Q_y, M_x for the plots.

1st section: $0 \le z_1 \le a \Rightarrow \quad Q_y(z_1) = R_A = F; \quad M_x(z_1) = R_A \cdot z_1;$

$$z_1 = 0 \Rightarrow M_x = 0; \quad z_1 = a \Rightarrow M_x = Fa.$$

2nd section: $a \le z_2 \le (\ell - a) \;\Rightarrow\; Q_y(z_2) = R_A - F = F - F = 0$

$M_x(z_2) = R_A z_2 - F(z_2 - a) = Fz_2 - Fz_2 + Fa = Fa..$

The shearing force on the section is zero, so $M_x =$const, the section undergoes **pure bending.**

3rd section: $0 \le z_3 \le a \;\Rightarrow\; Q_y(z_3) = -R_B = -F$; $M_x(z_3) = R_B \cdot z_3$;

$$z_3 = 0 \Rightarrow M_x = 0; \quad z_3 = a \Rightarrow M_x = Fa.$$

Example 4.

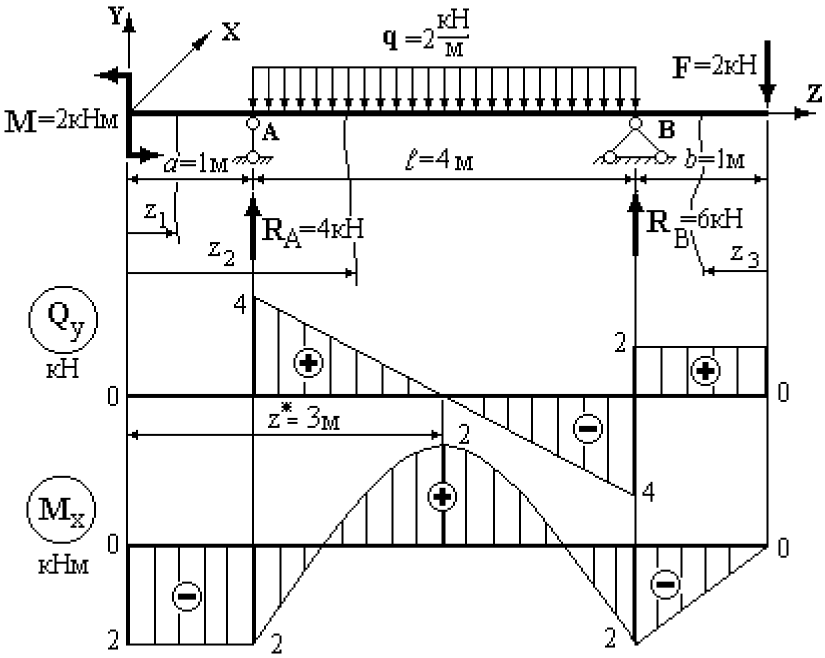


Figure 1.10 - Double cantilever beam

1. Reactions of supports.

$\sum M_A = 0; \quad M - q \cdot 4 \cdot 2 - F \cdot 5 + R_B \cdot 4 = 0; \quad 2 - 2 \cdot 4 \cdot 2 - 2 \cdot 5 + R_B \cdot 4 = 0,$

$R_B = 6$kN.

$\sum M_B = 0; \quad M + q \cdot 4 \cdot 2 - F \cdot 1 - R_A \cdot 4 = 0; \quad 2 + 2 \cdot 4 \cdot 2 - 2 - R_A \cdot 4 = 0;$

$R_A = 4$ kN.

Verification: $\sum F_y = R_A + R_B - q \cdot 4 - F = 6 + 4 - 2 \cdot 4 - 2 \equiv 0.$

3. Divide the beam into sections, show the sections on each of them, indicate the boundaries of change for z_i, determine and calculate the functions Q_y and M_x .

1st section: $0 \le z_1 \le 1 \Rightarrow \; Q_y(z_1) = 0; \quad M_x(z_1) = -2$ kNm.

2nd section: $1 \le z_2 \le 5 \Rightarrow \; Q_y(z_2) = R_A - q(z_2 - 1) = 4 - 2(z_2 - 1);$

$$M_x(z_2) = -M + R_A(z_2 - 1) - q(z_2 - 1)\cdot\frac{(z_2 - 1)}{2} = -2 + 4\cdot(z_2 - 1) - 2\cdot\frac{(z_2 - 1)^2}{2};$$

$z_2 = 1 \Rightarrow Q_y = 4$ kN; $M_x = -2$ kNm. $z_2 = 5 \Rightarrow Q_y = -4$ kN; $M_x = -2$ kNm.

The shearing force Q_y changes the sign, the bending moment M_x reaches the extreme - the maximum value at z^*, which is determined by the condition

$Q_y(z^*) = 4 - 2(z^* - 1) = 0,$ from where $z^* = 3$m, and

$M_{x\,\max} = -2 + 8 - 4 = 2$ kNm.

3rd section: $0 \le z_3 \le 1 \;\Rightarrow\; Q_y(z_3) = F = 2\,\kappa H; \quad M_x(z_3) = -Fz_3;$

$z_3 = 0 \Rightarrow M_x = 0; \; z_3 = 1 \Rightarrow M_x = -2$ kNm.

2. STRESS IN TRANSVERSE BENDING

In the case of direct transverse bending, a shearing force Q_y occurs in the cross section, which causes shear deformation, and a bending moment M_x, which causes bending deformation.

2.1. Normal stresses at pure bending

Consider the case of pure bending, when in the section there is only a bending moment.

We show the rod before deformation (Fig. 2.1, *a*) and after (Fig. 2.1, *b*) load bending moments M_x.

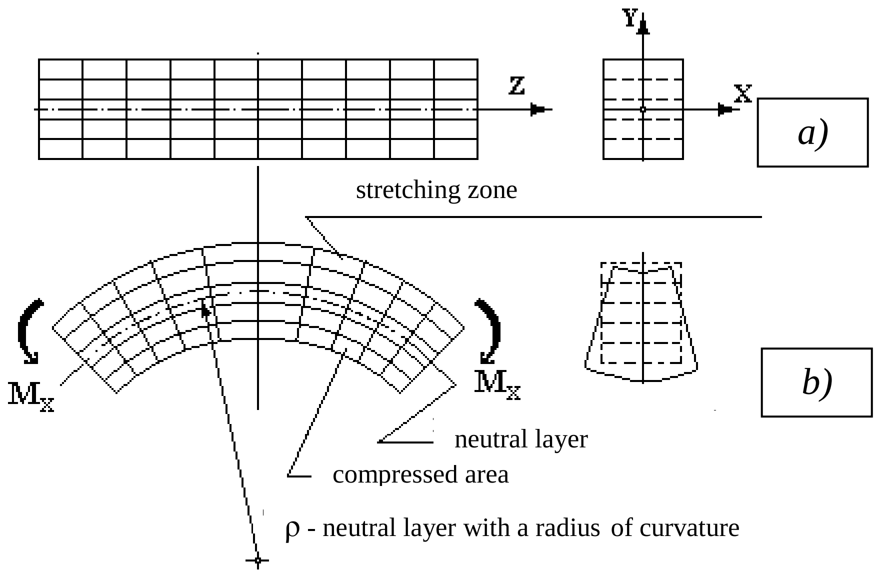


Figure 2.1 - Rod before deformation and after

Observing the deformation of the orthogonal grid, previously applied to the side surface of the beam before loading (Fig.2.1, *a*) and after (Fig. 2.1, *b*), we note that the longitudinal lines in pure bending are curved along the arc of the circle, the contours of cross sections remain flat, traces of which intersect longitudinal lines at right angles. In the compressed area (in this case at the bottom) the fibers shorten, in the stretching zone (at the top) lengthen.

There is a longitudinal layer, the length of which remains unchanged during pure bending. This layer is called neutral. The tensile zone and the compression zone in the beam are separated by a neutral layer with a radius of curvature ρ.

These circumstances allow us to introduce the following hypotheses. At pure bending **the hypothesis of flat sections** is observed. All cross-sections of the rod are not distorted during pure bending, but only rotate relative to each other around the X-axis. Longitudinal fibers do not press on each other. Normal stresses do not change along the width of the section.

It is logical to assume that at the points of cross section during pure bending there are only normal stresses that lead to an integral internal force factor - bending moment M_x.

Due to the lack of shearing forces in the direction of the Y axis, it is obvious that the tangential stresses are absent at the points of intersection.

Consider a rectilinear rod of arbitrary cross section with the axis of symmetry Y with pure bending (Fig.2.2, *a*). In the section with the coordinate z we apply the method of sections and get: $M_x = M$ (Fig.2.2, *b*).

In this section, the moment M_x arises as the sum of the moments from the distributed internal forces (normal stresses σ). Let's select an elementary area dA with coordinates x, y (Fig. 2.2, *c*). Let the Y axis be the main axis and the X axis coincide with the neutral longitudinal layer.

The problem of determining the internal force factors belongs to the class of statically indeterminate problems, so then we apply the scheme of solving statically indeterminate problems.

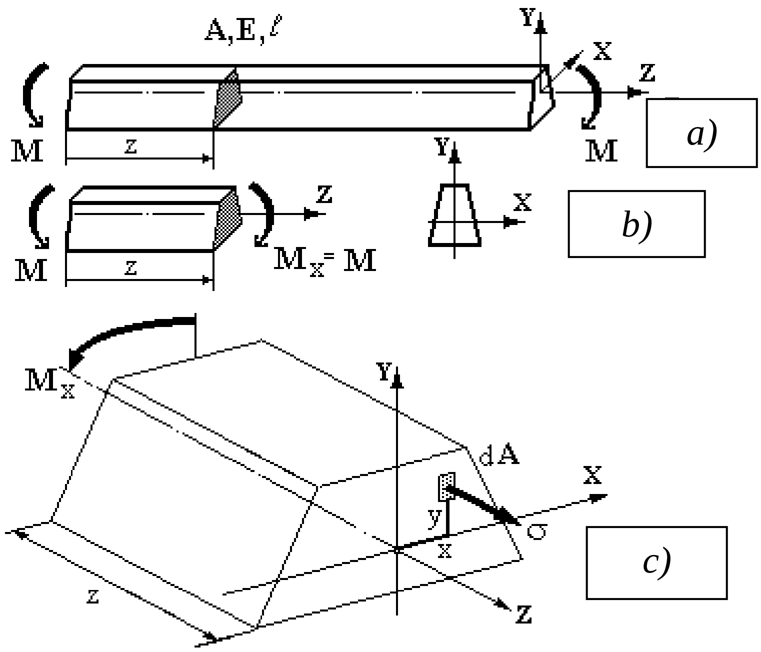


Figure 2.2 - Rectilinear rod of arbitrary cross section

Static side of the problem. Of the six equations of static equilibrium, three equations $\sum F_{ix} \equiv 0, \sum F_{iy} \equiv 0, \sum M_{iz} \equiv 0$ are performed identically. The elementary force in the axial direction acting on the area *dA*: $dN = \sigma dA,$ and the resulting force $N = \int_A \sigma dA.$ The elementary moment of force dN relative to the X and Y axes will be written as $M_x = \int_A dNy = \int_A \sigma y dA$ and $dM_y = dN \cdot x.$ Respectively bending moments: $M_x = \int_A dNy = \int_A \sigma y dA;\ M_y = \int_A dNx = \int_A \sigma x dA.$

Thus, the static conditions will take the form:

$$\sum F_{iz} = 0;\ \int_A \sigma dA = 0; \tag{2.1}$$

$$\sum M_{iy} = 0; \int_A \sigma x dA = 0; \tag{2.2}$$

$$\sum M_{ix} = 0; M_x - \int_A \sigma y dA = 0. \tag{2.3}$$

Note the unknown: normal stress σ - the magnitude and law of distribution;

ρ- radius of curvature of the neutral layer; position of neutral layer.

Geometric side of the problem. Consider the deformation of an element of length dz. Let the fiber OO_1 coincide with the neutral layer, select the fiber *ab* at a distance *y* from it (Fig. 2.3).

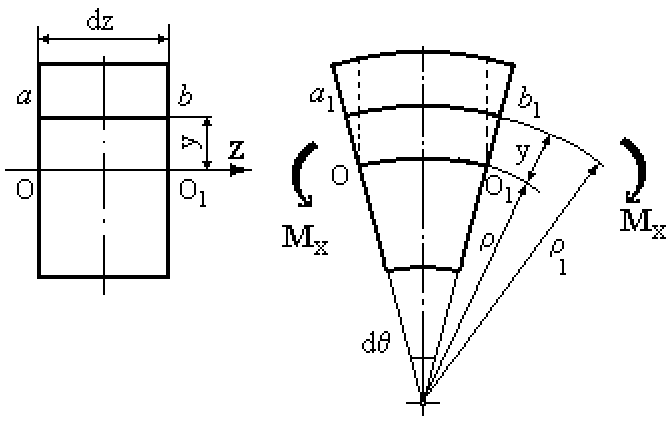


Figure 2.3 - Deformation of different fibers

The original length of the fiber is $\ell_0 = ab = OO_1 = \rho d\theta$, because the fiber OO_1 is not deformed. In the process of deformation, the length of the fiber a_1b_1 will be the length of the arc: $\ell_1 = \cup a_1b_1 = \rho_1 d\theta = (\rho + y)d\theta$. Determine the relative deformation of the fiber ab $\varepsilon_{\mathbf{ab}} = \frac{\Delta\ell_{\mathbf{ab}}}{\ell_0} = \frac{\ell - \ell_0}{\ell_0} = \frac{(\rho + y)d\theta - \rho d\theta}{\rho d\theta} = \frac{y}{\rho}$. Since the longitudinal fibers do not press on each other, then, apparently, this dependence occurs for any fiber:

$$\varepsilon = \frac{y}{\rho}. \tag{2.4}$$

This is an additional condition - the joint deformation equation in pure bending.

The physical side of the problem. In pure bending, the longitudinal fibers are subject to stretching and compression, so Hooke's law is valid for uniaxial stress $\sigma = E\varepsilon$.

After substituting the value of ε from expression (2.4) we have

$$\sigma = E\frac{y}{\rho}. \tag{2.5}$$

Substituting (2.5) sequentially into equations (2.1), (2.2), (2.3), we obtain the following.

1. $\int_A \sigma dA = \int_A E\frac{y}{\rho}dA = \frac{E}{\rho}\int_A ydA = \frac{E}{\rho}S_x = 0$ The modulus of longitudinal elasticity E for the material is a nonzero constant; the radius of curvature ρ of the neutral layer is a finite value. Thus, the static moment of the area $S_x = 0$. Therefore, the neutral layer at pure bending coincides with the central axis of the section, ie the y coordinate is calculated from the neutral line of the section - the geometric location of points at which the normal bending stresses are zero.

2. $\int\limits_A \sigma x dA = \int\limits_A E\frac{y}{\rho}xdA = \frac{E}{\rho}\int\limits_A xydA = \frac{E}{\rho}I_{xy} = 0$ If the centrifugal moment of inertia I_{xy} about the central axes is zero, then these axes are the main axes of inertia. Thus, the *XY* axes are the main axes of inertia and the neutral line is the main central axis of inertia, it is perpendicular to the plane of action of the load.

3. $M_x - \int\limits_A \sigma y dA = M_x - \int\limits_A E\frac{y}{\rho}ydA = M_x - \frac{E}{\rho}\int\limits_A y^2 dA = M_x - \frac{E}{\rho}I_x = 0$ whence the curvature of the neutral longitudinal layer is determined by the expression:

$$\frac{1}{\rho} = \frac{M_x}{EI_x}, \tag{2.6}$$

which is called the Navier equation. Here $\int\limits_A y^2 dA = I_x$ is the axial moment of inertia of the section, and EIx is the stiffness of the rod during bending.

Comparing the values of curvature $\frac{1}{\rho}$ from equations (2.5) and (2.6) we obtain:

$$\left\{\begin{matrix} \frac{1}{\rho} = \frac{\sigma}{Ey} \\ \frac{1}{\rho} = \frac{M_x}{EI_x} \end{matrix}\right\} \Rightarrow \frac{\sigma}{Ey} = \frac{M_x}{EI_x}.$$

The formula for determining normal stresses takes the form:

$$\sigma = \frac{M_x y}{I_x}. \tag{2.7}$$

From the obtained formula it follows that the normal stresses along the height of the section change linearly, because the bending moment M_x and the moment of inertia *Ix* of the cross section are constant. Figure 2.4 shows the distributions of normal stresses in height for different cross-sectional shapes.

Maximum stresses σ_{max} occur at the farthest points from the neutral line at $y = y_{max}$, that is

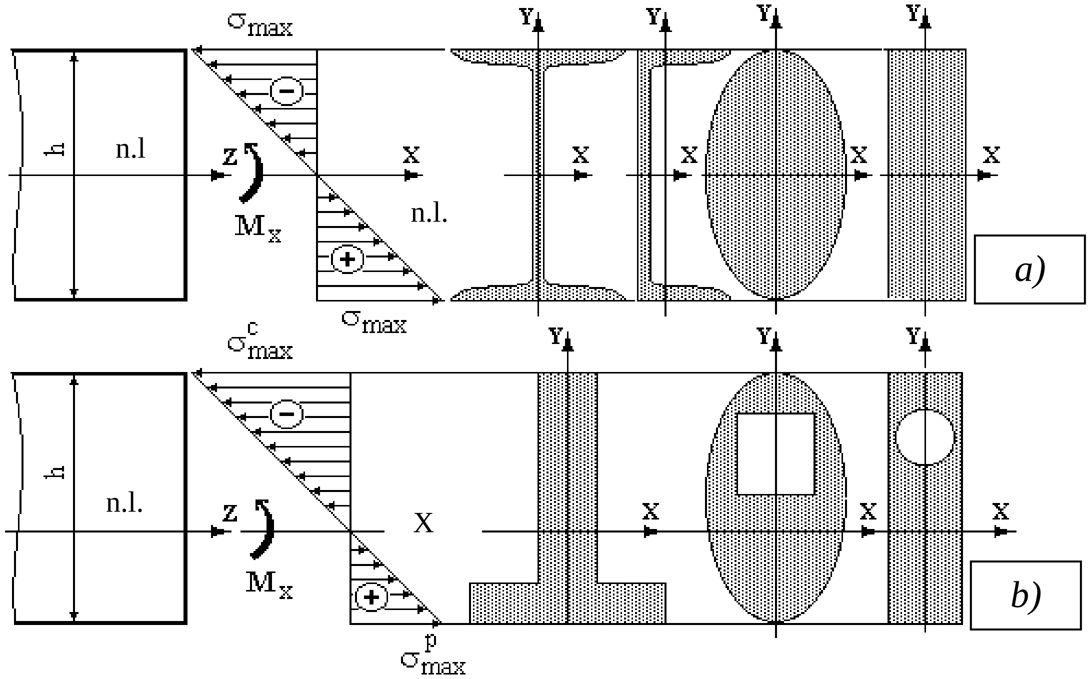


Figure 2.4 - Distributions of normal stresses in height for different cross-sectional shapes

$\sigma_{max} = \frac{M_x \cdot y_{max}}{I_x}$, which must be compared with the allowable stress $[\sigma]$.

Thus, the bending strength condition takes the form

$$\sigma_{max} = \frac{M_{x\,max} \cdot y_{max}}{I_x} \le [\sigma]. \qquad (2.8)$$

In practice, this form is used to calculate sections with one axis of symmetry (Fig. 2.4, *b*). Given the fact that $W_x = \frac{I_x}{y_{max}}$ - the axial moment of resistance, it is more convenient for sections with two axes of symmetry (Fig. 2.4, *a*) to use the condition of bending strength in the form:

$$\sigma_{max} = \frac{M_{x\,max}}{W_x} \le [\sigma]. \tag{2.9}$$

In the case of transverse bending, when the shearing force Q_y is not equal to zero, there is a curvature of the cross sections, and the hypothesis of flat sections is not true. Studies show that with respect to the length ℓ of the rod to the height *h* of the cross section $\frac{\ell}{h} \ge 8$ (for most beams) we can assume that the cross section is practically not curved, then formula (2.7) for determining normal stresses is valid for transverse bending.

Example. Determine the dimensions of different shapes of cross sections, if the bending moment in the cross section $M_x = 80$ kNm, the allowable bending stress $[\sigma] = 160$ MPa.

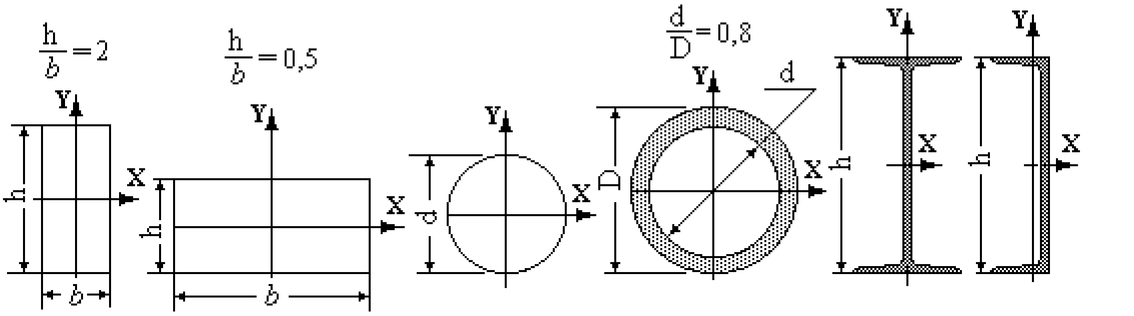
From the condition of strength $\sigma_{max} = \frac{M_{x\,max}}{W_x} \le [\sigma]$ section modulus of section

$W_x \ge \frac{M_{x\,max}}{[\sigma]} = \frac{80 \cdot 10^3}{160 \cdot 10^6} = 0{,}5 \cdot 10^{-3} м^3$ = 500 cm^3.

Next, design a section (Fig. 2.5).

1. **Rectangular section** (Fig. 2.5, *a*), for which the ratio h/b should be set (take $h/b = 2$). section modulus $W_x = \frac{bh^2}{6} = \frac{2}{3}b^3 = 500\,\text{cm}^3$, from where $b = \sqrt[3]{750} \approx 9$ cm. Height of section $h = 18\,\text{cm}$, cross-sectional area $A = bh = 2b^2 = 162\,\text{cm}^2$.

2. **Rectangular section** for which the ratio $\frac{h}{b} = \frac{1}{2}$ (Fig. 2.5, *b*). By analogy: the section modulus $W_x = \frac{bh^2}{6} = \frac{b^3}{24} = 500$ cm^3, from where $b = \sqrt[3]{12000} = 22{,}8$ cm, $h = 11{,}4$ cm, $A = 22{,}8 \cdot 11{,}4 = 260$ cm^2.



a) h=18cm b=9cm	b) h=11,4cm b=22,8cm	c) d=17,1cm	d) D=20,4cm d=16,32cm	e) I №30a	f)] №33	
162cm^2	260cm^2	229cm^2	115cm^2	50cm^2	47cm^2	A_i
3,24	5,2	4,58	2,3	≈1	≈1	A_{opt}/A_i

Figure 2.5 - Different shapes of cross sections

3. **Circle section** with a diameter of *d* (Fig. 2.5, *c*). Axial section modulus $W_x = \frac{\pi d^3}{32} \approx 0{,}1d^3 = 500$ cm^3, whence the diameter of the section $d = \sqrt[3]{5000} = 17{,}1$ cm, area $A = \frac{\pi d^2}{4} = \frac{\pi \cdot 17{,}1^2}{4} = 229$ cm^2.

4. **Circular hollow section** (Fig. 2.5, *d*). Set by the ratio of diameters $\alpha = \frac{d}{D}$, section modulus $W_x = \frac{\pi D^3}{32}(1-\alpha^4) \approx 0{,}1D^3(1-\alpha^4) = 500$ cm^3. Let $\alpha = 0{,}8$,

then $D = \sqrt[3]{\frac{5000}{1-0{,}8^4}} = 20{,}4$ cm, $d = 20{,}4 \cdot 0{,}8 = 16{,}32$ cm, area

$A = \frac{\pi D^2}{4}(1-\alpha^2) = \frac{\pi \cdot 20{,}4^2}{4}(1-0{,}8^2) = 115$ cm^2.

5. **I-beam section** (Fig. 2.5, *e*). Select the I-beam number with the nearest larger value of the section modulus to the calculated value. For the I-beam № 30a: $W_x = 518$ cm^3, $A \approx 50$cm^2.
6. **Channel section** (Fig. 2.5, *f*). Select the channel section beam number with the nearest larger value of the section modulus to the calculated value. For the channel section beam № 33 $W_x = 484$cm^3, $A \approx 47$cm^2.

Taking the ratio of individual areas to the area of the rational cross section (I-beam, channel), we obtain the coefficient of material consumption. Let's make the table (Fig. 2.5) from which it follows that the most rational are I-beam and channel sections in which the smallest area of cross section and the smallest expense of material.

2.2. Tangential stresses at transverse bending

The shear force in the cross section causes tangential stresses τ, which coincide in the direction with it, do not change along the width of the cross section and are determined by the formula of D. Zhuravsky:

$$\tau = \frac{Q_y S_x^{cut}}{b_y \cdot I_x} , \qquad (2.10)$$

where Q_y is the shearing force acting in the cross section; I_x - axial moment of inertia (second area moment) of the section relative to the Central axis *X* (neutral line); b_y - section width at the level y from the neutral line where the tangential stresses are determined; $S_x^{cut} = A^{cut} \cdot y_c$ - the absolute value of the static moment relative to the central axis *X* of the part of the section that lies above or below the level where the tangential stresses are determined. The strength condition of tangential stress in transverse bending is written in the form:

$$\tau_{\max} = \frac{Q_{y\max} S_{x\ \max}^{cut}}{b_y \cdot I_x} \leq [\tau] \quad . \tag{2.11}$$

Thus, with direct transverse bending we have the conditions of strength for normal σ and tangential τ stresses. The main condition is the strength of normal stresses, and condition (2.11) of tangential stresses, as a rule, is checked. The use of D. Zhuravsky's formula will be analyzed by examples.

2.3. Distribution of tangential stresses for a rectangular section

In the cross section there are *Mx* moment and shear force Q_y, directed as shown in Fig.2.6. Shear force Q_y, section width $b_y = b$ and axial moment of inertia $I_x = \frac{bh^3}{12}$ are specific constants (Fig. 2.6). Thus, the tangential stresses vary according to the same law as the static moment of the cut-off part of the area S_x^{cut}.

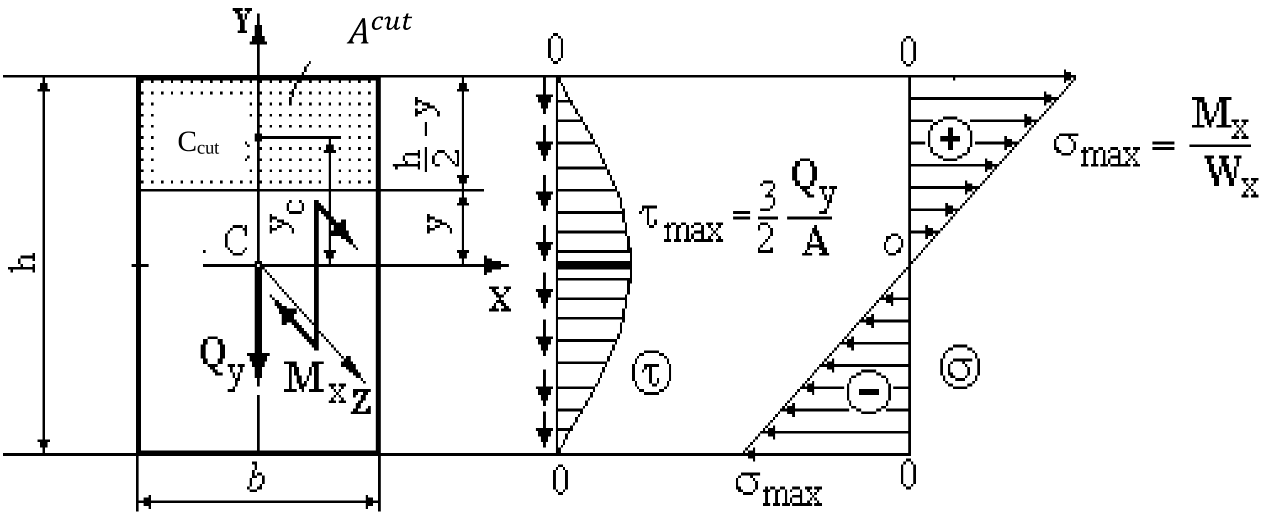


Figure 2.6 – Tangential and normal stresses for rectangular section

Determine the tangential stresses at level *y*. The area of the cut part of the section $A^{cut} = b \cdot \left(\frac{h}{2} - y\right) = \frac{bh}{2}\left(1 - \frac{2y}{h}\right)$, position of its center of gravity $y_c = \frac{h}{2} - \frac{\frac{h}{2} - y}{2} = \left(\frac{h}{4} + \frac{y}{2}\right) = \frac{h}{4}\left(1 + \frac{2y}{h}\right)$. Static moment of the cut off part of the area: $S_{\tilde{o}}^{cut} = A^{cut} \cdot y_c = \frac{bh}{2}\left(1 - \frac{2y}{h}\right) \cdot \frac{h}{4}\left(1 + \frac{2y}{h}\right) = \frac{bh^2}{8}\left[1 - \left(\frac{2y}{h}\right)^2\right]$.

Thus, the tangential stresses vary according to the law of the quadratic parabola. The maximum tangential stresses occur on the neutral line, where the normal stresses σ are zero. To determine $\tau_{\max}$ it is necessary to calculate the static moment of half the cross-sectional area $S_{x\max}^{cut}$, and the maximum tangential stresses will be defined as:

$$\tau_{\max} = \frac{Q_{\acute{o}} S_{x\ \max}^{cut}}{b_y \cdot I_x}.$$

For a rectangular cross section $b_y = b$, $I_x = \frac{bh^3}{12}$, $S_{x\ \max}^{cut} = \frac{bh^2}{8}$ we have:

$$\tau_{\max} = \frac{Q_y\, bh^2 / 8}{b \cdot bh^3 / 12} = \frac{3}{2}\frac{Q_y}{bh} = \frac{3}{2}\frac{Q_y}{A}.$$

2.4. Distribution of tangential stresses for I-beam section

In the cross section there are *Mx* moment and shear force Q_y, directed as shown in Fig. 2.7.

Using expression (2.10) for tangential stresses, we determine their values at characteristic points.

Point 1: $\tau_1 = 0$ because $S_x^{cut} = 0$ (above level 1 the truncated area is missing).

Points 2,3. These points have the same *y* coordinate, but belong to the shelf and the wall at the same time, ie different widths $b_2=b$; $b_3=d$. Therefore, in the place of transition of the shelf into the wall there is a jump of tangential stresses.

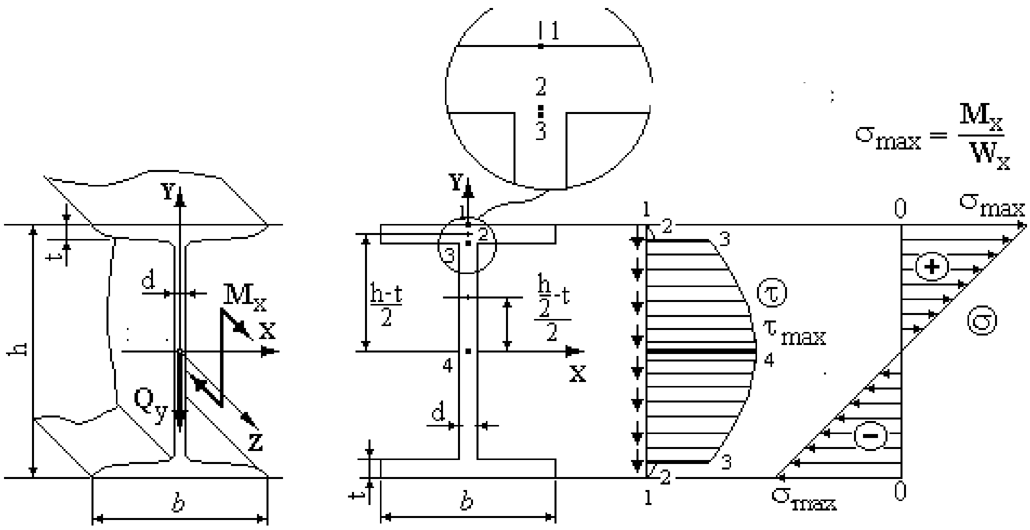


Figure 2.7 - Tangential and normal stresses for I-beam section

Point 2 (belonging to the shelf): $\tau_2 = \frac{Q_y}{b \cdot I_x} bt\left(\frac{h-t}{2}\right) = \frac{Q_y t}{I_x}\left(\frac{h-t}{2}\right)$;

Point 3 (belonging to the wall): $\tau_3 = \frac{Q_y}{d \cdot I_x} bt\left(\frac{h-t}{2}\right)$.

Point 4: $\tau_4 = \frac{Q_y}{bI_x}\left[bt\left(\frac{h-t}{2}\right) + d\left(\frac{h}{2} - t\right) \cdot \left(\frac{\frac{h}{2} - t}{2}\right)\right] = \frac{Q_y S_{x\max}^{cut}}{d \cdot I_x}$.

$S_{x\max}^{cut}$ - static moment relative to the central axis of half the cross-sectional area, for standard profiles are given in the assortment tables. An exemplary graph of tangential stress distribution is shown in Figure 2.7. The actual distribution of tangential stresses is slightly different from that obtained, because the shelves have slopes, and the transition from the shelf to the wall is carried out along the radius of curvature.

2.5. Performing design calculation

1. From the condition of strength by normal stresses we determine the section modulus of the cross section, ie $W_x \geq \frac{M_{x\max}}{[\sigma]}$, and design the cross section.

2. Check the cross section for tangential stresses. If $\tau_{\max} \leq [\tau]$ so, the calculation is complete. If $\tau_{\max} > [\tau]$ (exceeding by more than 5%), the cross-sectional dimensions are determined from the condition of tangential stress. There is no need to check the cross section for normal stresses, because its dimensions will be larger.

Example 1. For this scheme of loading the beam (Fig. 2.8) to determine the dimensions of the I-beam cross section, if the allowable normal stress $[\sigma]=150$ MPa, tangent - $[\tau]=100$ MPa.

Determine the reactions:

$\sum M_A = Fa + F(\ell + a) - R_B\ell = 0,\ R_B = 45\text{kN};$

$\sum M_B = F(\ell + a) + Fa - R_A\ell = 0,\ R_A = 45\text{kN}.$

Check: $\sum F_y = -F + F + R_A - R_B \equiv 0$.

1. Divide the beam into three segments, write for the current section on each section of the expression (function) Q_y and M_x:

$0 \leq z_1 \leq a\ \ Q_y = -F = 30$ kN; $M_x = -F \cdot z_1$;

$0 \leq z_2 \leq \ell\ \ Q_y = -F + R_A = +15$ kN; $M_x = -F(z_2 + a) + R_A \cdot z_2$;

$0 \leq z_3 \leq a\ \ Q_y = -F = -30$ kN; $M_x = -F \cdot z_3$.

We calculate Q_y, M_x in characteristic sections and build plots.

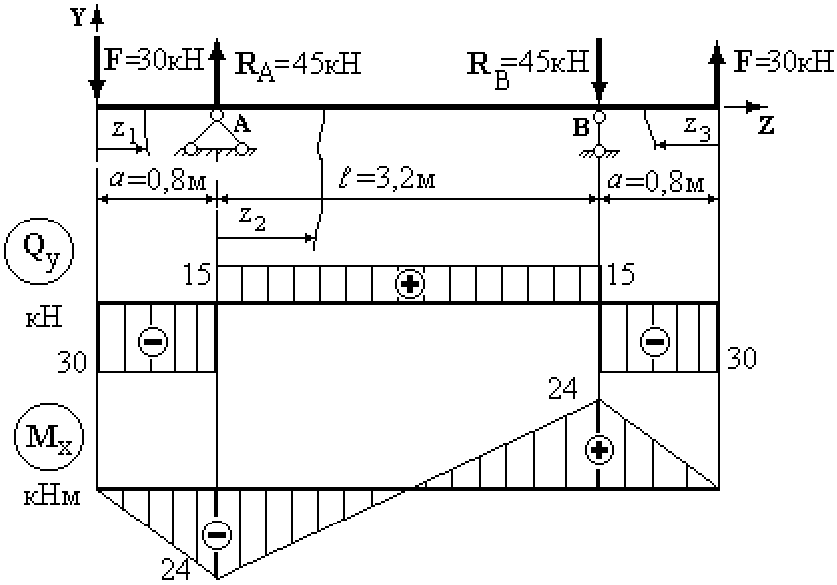


Figure 2.8 – Plots Q_y and M_x

2. From the condition of strength at normal stresses (where $M_{x\max}$), a dangerous cross section is on the supports; from the condition of tangential stress (where $Q_{y\max}$) any section on the consoles is equally dangerous. So, $M_{x\max}$ = 24 kNm; $Q_{y\max}$ = 30 kN.

3. From the condition of strength by normal stresses we determine the section modulus: $W_x \geq \frac{M_{x\max}}{[\sigma]} = \frac{24 \cdot 10^3}{150 \cdot 10^6} = 0{,}16 \cdot 10^{-3}$ m^3 = 160 cm^3.

We choose an I-beam №18a cm^3, which is slightly less than the calculated value.

Other parameters necessary for calculation: $A = 25{,}4\,\text{cm}^2$, $I_x = 1430\,\text{cm}^4$ = $1430 \cdot 10^{-8}$ m^4; $d = 5{,}1$ mm = $5{,}1 \cdot 10^{-3}$ m, $S_{x\max}^{\text{відс}} = 89{,}8$ m^3= $89{,}8 \cdot 10^{-6}$ m^3 .

Check the cross section for tangential stresses:

$$\tau_{\max} = \frac{Q_y S_{x\max}^{\text{відс}}}{d \cdot I_x} = \frac{30 \cdot 10^3 \cdot 89{,}8 \cdot 10^{-6}}{5{,}1 \cdot 10^{-3} \cdot 1430 \cdot 10^{-8}} = 36{,}9 \cdot 10^6 \text{ N/m}^2 = 36{,}9 \text{ MPa} < [\tau],$$

the strength condition is met and the calculation is completed.

Example 2. For this scheme of loading the wooden beam (Fig. 2.9) determine the dimensions of the rectangular section, if the ratio of the sides $\frac{h}{b}=2$, the allowable normal stress $[\sigma]=10$ MPa, tangent $[\tau]=2{,}5$ MPa.

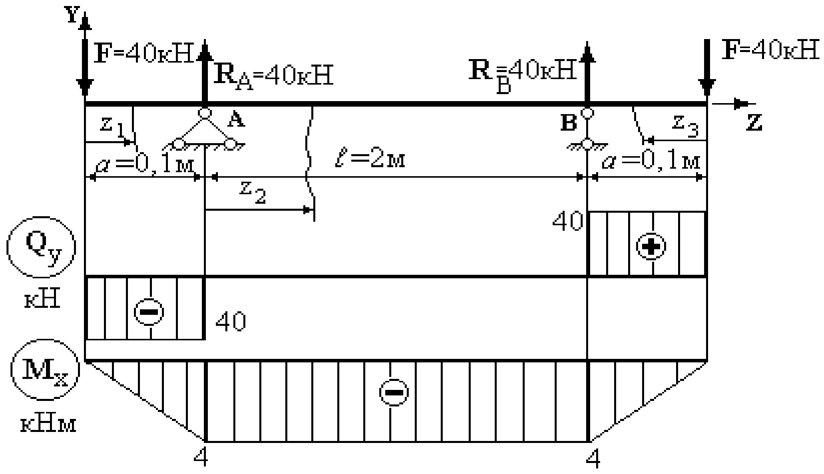


Figure 2.9 - Double cantilever beam

Since the load is symmetrical, the reference reactions are the same and equal to half the external load, thus $R_A = R_B = 40$ kN.

1. Determine the shear forces and bending moments in sections.

1st segment: . $0 \le z_1 \le a \Rightarrow Q_y = -F = -40$ kN; $M_x = -F \cdot z_1$.

2nd segment: $0 \le z_2 \le \ell \Rightarrow Q_y = -F + R_A = 0$;

$$M_x = -F \cdot (z_2 + a) + R_A \cdot z_2 = -40 z_2 - 40 \cdot a + 40 z_2 = -40 \cdot a = -4 \text{ kNm}.$$

3rd segment: $0 \le z_3 \le a \Rightarrow Q_y = F = 40$ kN; $M_x = -F \cdot z_3$.

On the received functions we construct plots Q_y and M_x. From the condition of tangential stress, any cross-section on the consoles is equally dangerous, and from the condition of normal stress, it is equally dangerous. any section on the span of the beam.

2. From the condition of strength at normal stresses the section modulus :

$$W_x \ge \frac{M_{x\max}}{[\sigma]} = \frac{4 \cdot 10^3}{10 \cdot 10^6} = 0{,}4 \cdot 10^{-3} \text{m}^3 = 400 \text{ cm}^3; \; W_x = \frac{bh^2}{6} = \frac{2}{3} b^3 = 400 \text{cm}^3,$$

from where : $b = \sqrt[3]{\frac{3}{2} \cdot 400} = 8{,}4\,\text{cm},\ h = 2b = 16{,}8\,\text{cm},$

$A = bh = 2b^2 = 2 \cdot 8{,}4^2 = 142\ \text{cm}^2$

3. The maximum tangential stress for a rectangular section is: $\tau_{\max} = \frac{3}{2}\frac{Q_{y\max}}{A} = \frac{3 \cdot 40 \cdot 10^3}{2 \cdot 142 \cdot 10^{-4}} = 4{,}22 \cdot 10^6\,\text{N/m}^2 = 4{,}22\ \text{MPa} > [\tau] = 2{,}5\ \text{MPa}$,

   the strength condition is not met.

Determine the dimensions of the cross section from the condition of tangential stress: $\tau_{\max} = \frac{3}{2}\frac{Q_{y\max}}{A} \le [\tau]$, from which we find the cross-sectional area:

$$A = \frac{3Q_{y\max}}{2[\tau]} = \frac{3 \cdot 40 \cdot 10^3}{2 \cdot 2{,}5 \cdot 10^6} = 24 \cdot 10^{-3}\,\text{m}^2 = 240\,\text{cm}^2$$

Area $A = bh = 2b^2 = 240\,\text{cm}^2$, whence the width of the section: $b = \sqrt{120} \approx 11\,\text{cm}$, and height $h = 22\,\text{cm}$.

From the condition of tangential stress, the cross-sectional dimensions are larger than from the condition of normal stress.

2.6. Potential energy of deformation during bending

Pure bending ($Q_y = 0, M_x \neq 0$). ***The potential energy of deformation during pure bending is determined by the work of internal bending moments on the angular displacement of the section.***

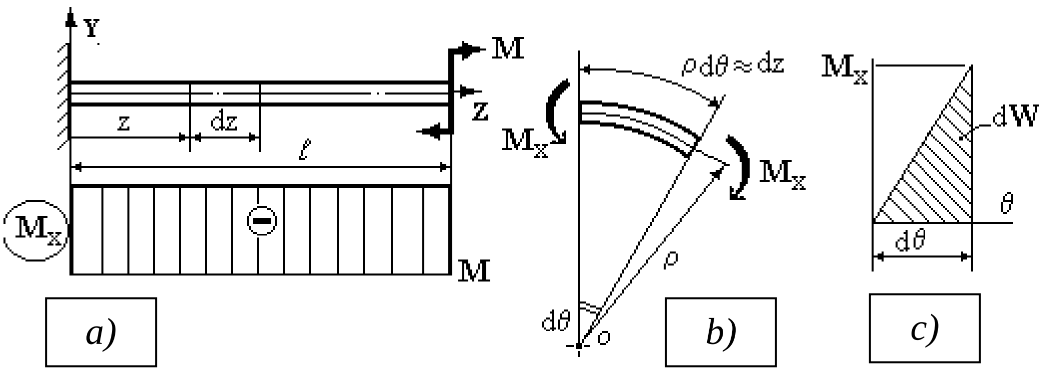


Figure 2.10 - Pure bending of beam

Consider the rod with pure bending (Fig. 2.10, *a*). Select the element of the rod length dz (Fig. 2.10, *b*). Under static load, the neutral axis is curved along the radius ρ of the circle, the extreme sections are rotated by an angle $d\theta$. Within the framework of Hooke's law, the dependence between the moment M_x and the angle of rotation $d\theta$ under static load is linear (Fig. 2.10, *c*). The elementary work of internal forces is determined by the area of the triangle, ie $dW = \frac{1}{2} M_x d\theta$.

But the work is numerically equal to the potential energy of deformation dU, ie. $dU = \frac{1}{2} M \cdot d\theta$. From Fig.2.10b it follows that $d\theta = \frac{dz}{\rho}$, thus, $dU = \frac{1}{2} M_x \frac{dz}{\rho}$. The curvature of the neutral axis $\frac{1}{\rho} = \frac{M_x}{EI_x}$, then $dU = \frac{M_x^2 dz}{2EI_x}$. The total potential energy of the rod is the integral of the length of the rod:

$$U = \int_{\ell} \frac{M_x^2(z)dz}{2EI_x} .. \tag{2.12}$$

Transverse bending ($Q_y \neq 0, M_x \neq 0$). As shown by calculations for rods in which the ratio of length ℓ to section height h is greater then $8 \div 10$ ($\frac{h}{\ell} > 8 \div 10$), the potential energy of deformation from the shear force U_{Q_y} is $0{,}4 \div 0{,}5\%$ the potential energy of deformation of the bending moment U_{M_x}. Therefore, when determining the potential energy of deformation during bending, only the potential energy of deformation from the bending moment M_x, which is determined by expression (2.12), is taken into account (2.12).

3. DISPLACEMENT IN STRAIGHT BENDING.

3.1. Differential equation of a curved axis

We obtain the differential equation of the curved axis with straight bending (the plane of action of the loads coincides with one of the main axes of inertia). The rectilinear axis of the beam under the action of external loads (Fig. 3.1) is transformed into a flat smooth curve and is called an elastic line (curved axis of the beam).

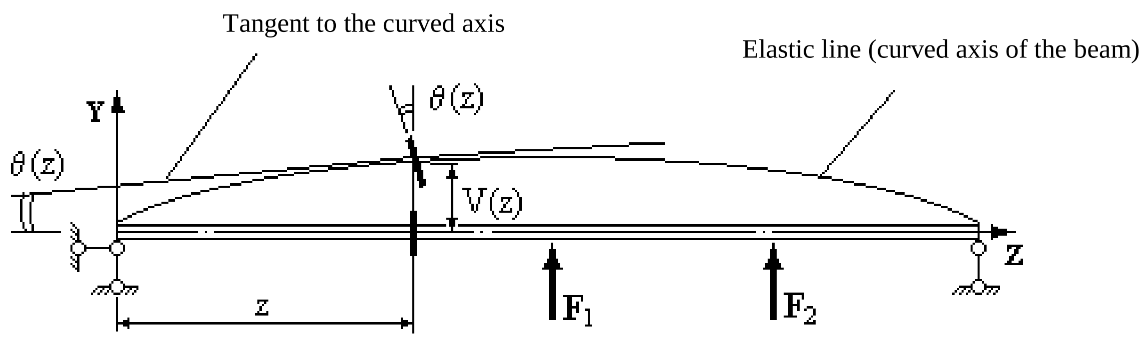


Figure 3.1 - Curved axis of the beam

The deflection of the beam $V(z)$ is the movement of the center of gravity of the section along the normal to the original axis. The maximum deflection is called the deflection arrow and is denoted by *f*. The section rotation angle $\theta(z)$ is the section rotation relative to the original position.

The tangent of the angle of inclination tangent to the curved axis is the first derivative of the function $V(z)$: $\mathrm{tg}\,\theta(z) = \frac{dV(z)}{dz} = V'(z)$. For small angles (tg $\theta(z) \approx \theta(z)$) the equation of rotation angles can be written as: $\theta(z) = V'(z)$.

The differential equation of the curved axis of the beam is obtained using the Navier equation, in which the curvature of the neutral axis during bending is defined as: $\frac{1}{\rho} = \frac{M_x}{EI_x}$. On the other hand, from the course of analytical geometry it is known that the curvature of a flat curve is defined as: $\frac{1}{\rho} = \pm \frac{V''(z)}{\{[1+[V'(z)]^2\}^{\frac{3}{2}}}$.

Comparing the right-hand sides of these two dependences, we obtain a nonlinear differential equation with respect to deflection $V(z)$:

$$\frac{M_x(z)}{EI_x} = \pm \frac{V''(z)}{\{1+[V'(z)]^2\}^{\frac{3}{2}}} \quad . \tag{3.1}$$

For small displacements (within elastic deformations), when, for example, $\mathrm{tg}\,\theta(z) = V'(z) \le 0{,}01$, the square of the first derivative in comparison with the unit can be neglected. Taking into account that the signs of the second derivative $V''(z)$ and the bending moment M_x coincide, we obtain a differential equation of the second order, which is called the differential equation of the curved axis of the beam for small displacements:

$$EI_x V''(z) = M_x(z). \tag{3.1, a}$$

Consecutively integrate twice and obtain the equation for angles of rotation and deflection:

$$EI_x V'(z) = EI_x \theta(z) = \int M_x(z)dz + C_1 , \tag{3.2}$$

$$EI_x V(z) = \int dz \int M_x(z)dz + C_1 z + C_2 , \tag{3.3}$$

where C_1 and C_2 are arbitrary constant integrations determined from boundary conditions.

Example 1. Consider a cantilever beam loaded on the free end with a concentrated force (Fig. 3.2).

Bending moment in cross section z: $M_x(z) = -Fz$. Write the differential equation of the elastic line of the beam: $EI_x V''(z) = -F \cdot z$. Integrating this equation twice, we obtain in accordance with (3.2), (3.3):

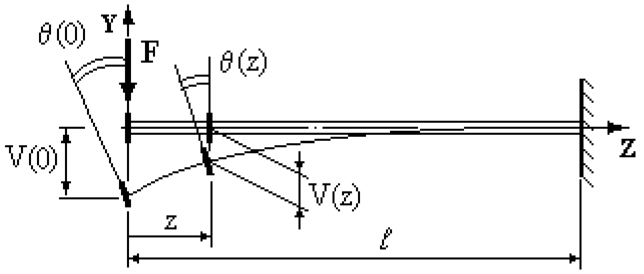


Figure 3.2 - Cantilever beam

$$EI_x V'(z) = \frac{-F \cdot z^2}{2} + C_1;$$

$$EI_x V(z) = \frac{-F \cdot z^3}{6} + C_1 z + C_2.$$

We will write down and fulfill the boundary conditions. When $z = \ell$ the angle of rotation $V'(\ell) = \theta(\ell) = 0$, ie, $-\frac{F \cdot \ell^2}{2} + C_1 = 0$ where: $C_1 = \frac{F\ell^2}{2}$. When $z = \ell$ deflection $V(\ell) = 0$, ie: $-\frac{F \cdot \ell^3}{6} + \frac{F \cdot \ell^2}{2}\ell + C_2 = 0$, from where: $C_2 = \frac{F \cdot \ell^3}{6} - \frac{F \cdot \ell^3}{2} = -\frac{F\ell^3}{3}$.

Taking into account the values C_1 and C_2 equations of the elastic line and angles of rotation will be written as:

$$EI_x V(z) = -\frac{Fz^3}{6} + \frac{F\ell^2}{2} z - \frac{F\ell^3}{3}; \; EI_x \theta(z) = -\frac{Fz^2}{2} + \frac{F\ell^2}{2}.$$

The largest deflection and angle of rotation occur at the origin when $z = 0$:

$$EI_x V(0) = -\frac{F\ell^3}{3}, \text{ from where: } |V(0)| = V_{\max} = f = \frac{F\ell^3}{3EI_x};$$

$$EI_x \theta(0) = \frac{F\ell^2}{2}, \text{ from where: } \theta(0) = \theta_{\max} = \frac{F\ell^2}{2EI_x}.$$

The maximum deflections $V_{\max}$ of the beams must be compared with the permissible deflection $[V]$. Then the condition of rigidity when bending the cantilever beam will take the form:

$$V_{\max} = f = \frac{Fl^3}{3EI_x} \le [V]. \qquad (3.4)$$

From here the axial moment of inertia $I_x \ge \frac{F\ell^3}{3E[V]}$, on the basis of what we design a section is defined. The allowable deflection is selected depending on the responsibility of the structure from the range $[V] = (\frac{1}{100} \div \frac{1}{1000})\ell$, where ℓ is the span of the beam.

Direct integration of the differential equation of an elastic line is cumbersome even in simple cases. Therefore, to determine the displacements in the beams, energy methods are more accepted, which lead to simple dependences.

3.2. Energy methods for determining displacements

We introduce the notation and basic concepts.

The bending moment from the external load $M_{xF}(z) = M_{xF}$ is denoted as M_F. **Bending moment from a unit force (moment)** - $\overline{M}_x$ or $\overline{M}$. **Displacement (deflection, angle of rotation) from the external load** is indicated Δ_{ij} where the first index *i* is related to the point or direction of movement; the second index j is related to the reason that caused the movement. **Linear displacement (deflection) from a unit force and angular displacement from a unit moment** are denoted δ_{ij}, where the index *i* is the point of the beam and the direction of movement; index *j* - the reason that caused a single movement.

3.2.1. *Maxwell – Mohr integral*

Consider a beam of a length ℓ loaded at point 1 by force F (Fig. 3.3). Determine the displacement Δ_{21} (at point 2 of the force applied at point 1).

1. The first condition. At point 1 we apply a concentrated force *F*. The deflection at point 1 is Δ_{11}, at point 2 is Δ_{21}. In sections of a beam there is a bending moment from external loading M_{xF}. The force *F* is applied statically and performs work $W_1 = \frac{1}{2} \cdot F \cdot \Delta_{11}$ on the way Δ_{11} (see the graph in Fig.3.3.1). Determine the potential energy of deformation, expressed in terms of bending moment, by the formula (3.12): $U_1 = \int_\ell \frac{M_{xF}^2 dz}{2EI_x}$. But the potential energy of deformation U_1 is numerically equal to the work of external forces W_1, ie: $W_1 = U_1$.

2. The second state. At point 2, we statically apply a unit force that, bending the beam, performs the work (see graph in Fig.3.3.2) on the displacement δ_{22}. In sections of a beam there is a bending moment $\overline{M}_x$ from unit force. The work of a unit force is $W_2 = \frac{1}{2} \cdot 1 \cdot \delta_{22}$. Potential deformation energy is $U_2 = \int_\ell \frac{\overline{M}_x^2 dz}{2EI_x}$. As in the previous case $W_2 = U_2$.

3. The third state. At point 2, we statically apply a unit force that, deforming the beam, performs the work W_2 on the displacement δ_{22} (see graph in Fig. 3.3.3). To the deformed beam statically in point 1 we apply the concentrated force F which, deforming a beam with already applied unit force, carries out work W_1 (see the schedule) on displacement Δ_{11}.

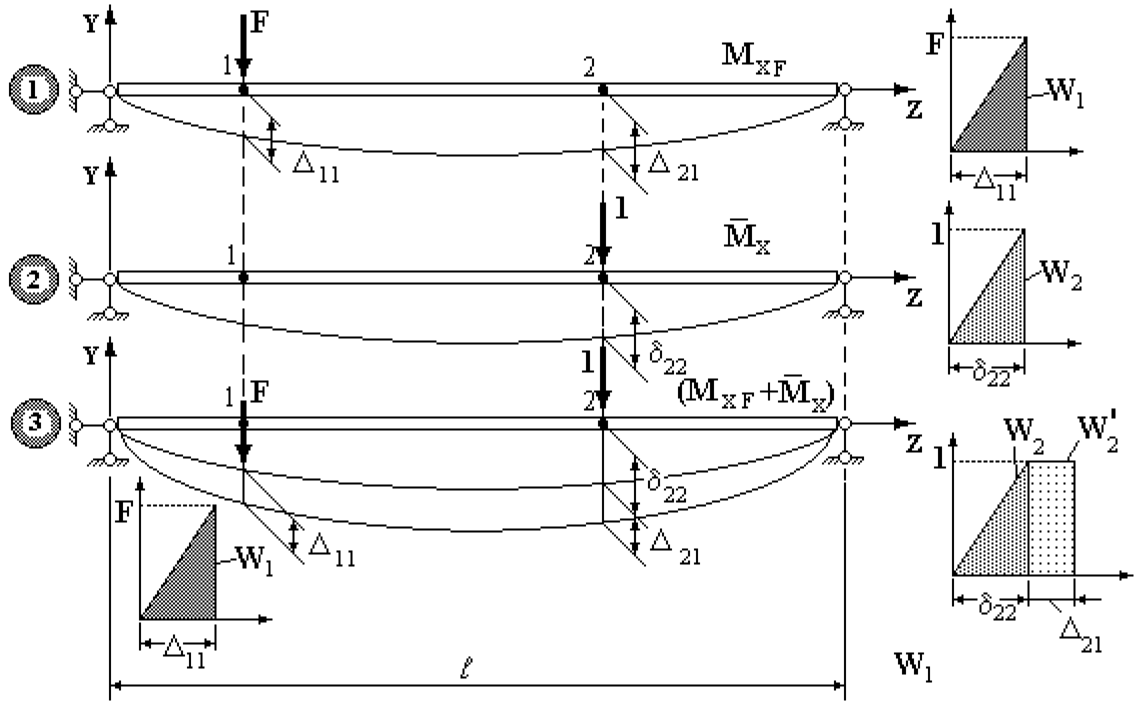


Figure 3.3 -Three conditions of beam

Point 2 will receive another displacement Δ_{21}, and a unit force will perform the work $W_2^* = 1 \cdot \Delta_{21}$ (see graph) on the displacement Δ_{21}. From action of force F and unit loading in sections of a beam there is a total bending moment $(M_{xF} + \overline{M}_x)$. The work of the two forces will be defined as:

$$W_3 = W_1 + W_2 + W_2^* = \frac{1}{2} F \cdot \Delta_{11} + \frac{1}{2} \cdot 1 \cdot \delta_{22} + 1 \cdot \Delta_{21},$$

and the potential energy of elastic deformation is expressed in terms of the total bending moment as:

$$U_3 = \int_\ell \frac{(M_{xF} + \overline{M}_x)^2 dz}{2EI_x} = \int_\ell \frac{M_{xF}^2 dz}{2EI_x} + \int_\ell \frac{\overline{M}_x^2 dz}{2EI_x} + \int_\ell \frac{M_{xF} \overline{M}_x}{EI_x} dz .$$

Comparing the expressions for W_3, U_3, after simple transformations we obtain:

$$\Delta_{21} = \int_\ell \frac{M_{xF} \overline{M}_x}{EI_x} dz . \quad (3.5)$$

The procedure for determining displacements using the Maxwell–Mohr integral.

1. We apply external loading, we define basic reactions, we break a beam into sites, we write down expressions (functions) of the bending moment M_{xF} for each site.
2. At the point, the movement of which is determined, apply:
   a) Unit force in determining the deflection (linear displacement);
   b) The unit moment in determining the angular displacement.

Determine the support reactions and in the same order as for the external load, write the expressions (functions) of the bending moment $\overline{M}_x$ at each section.

3. Substitute the functions (expressions) M_{xF}, $\overline{M}_x$ in the Maxwell-Mohr integral and make the appropriate calculations.
4. The result of the calculations is positive if the direction of the applied unit load coincides with the direction of actual movement, and negative if the direction of the applied unit load does not coincide with the direction of actual movement.

Example 2. Cantilever beam of constant cross section (EI_x=const) length ℓ loaded at the end of the concentrated force F (Fig. 3.4, *a*). Determine the deflection V_A and angle of rotation θ_A at the end of the console.

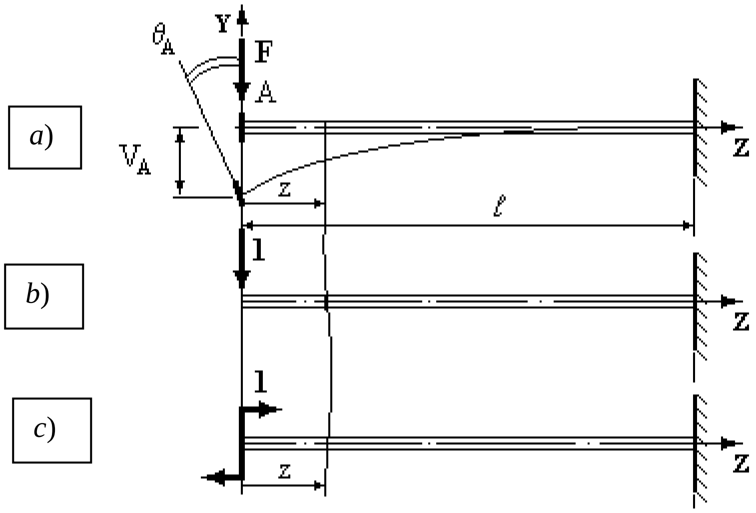


Figure 3.4 – Algorithm for determining displacements

1. Write the function $M_{xF} = -Fz$ (Fig. 3.4, *a*).
2. At a point A, apply a unit force (Fig. 3.4, *b*) and write the function $\overline{M}_x = -1 \cdot z$.
3. Substituting M_{xF}, $\overline{M}_x$ into the integral, we obtain: $V_A = \frac{1}{EI_x} \int_0^{\ell} F \cdot z \cdot z \cdot dz = \frac{F\ell^3}{3EI_x}$ (see example 1 in Fig. 3.2).
4. To determine the angular displacement at a point A, apply a unit moment (Fig. 3.4, *c*) and write the function $\overline{M}_x = 1$.
5. Substituting M_{xF}, M_{xF} into the integral, we obtain: $\theta_A = \frac{1}{EI_x} \int_0^{\ell} (-F \cdot z) \cdot 1 \cdot dz = -\frac{F\ell^2}{2EI_x}$.

The result of the calculation of the deflection V_A is positive, because the applied unit force coincides with the direction of the actual movement. The result of calculating the angle of rotation θ_A is negative, because the applied unit moment in the direction does not coincide with the actual direction of the angle of rotation of the section at the point A.

3.2.2. *Geometric method of calculating the Maxwell-Mohr integral. The method of multiplying plots*

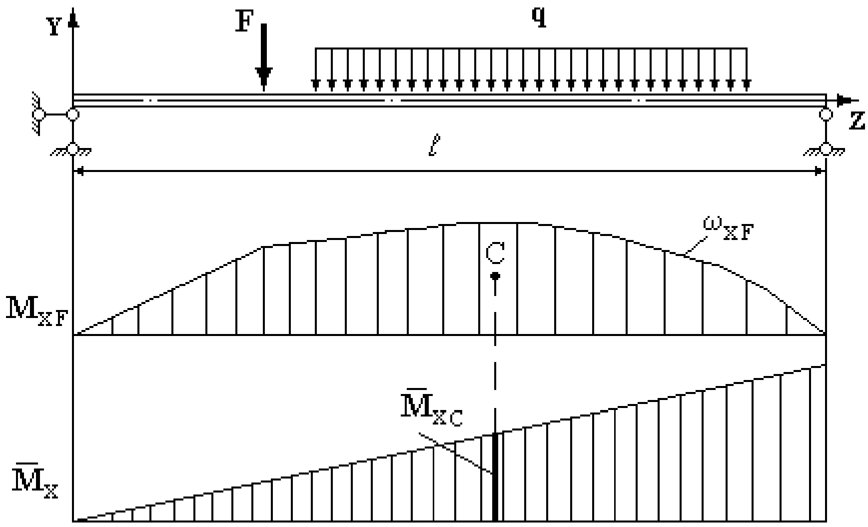


Figure 3.5 - Multiplying plots due to Vereshchagin's rule

<u>Vereshchagin's rule</u>. Using the geometric interpretation of the definition of the integral as the value of the area, the Mohr integral to determine the displacements in the beams of constant cross section can be calculated using a special operation on the plots of the corresponding bending moments.

As a result, we obtain:

$$\Delta_{ij} = \int_{\ell} \frac{M_{xF} \overline{M}_x}{EI_x} dz = \frac{\omega_{xF} \cdot \overline{M}_{xC}}{EI_x}, \tag{3.6}$$

where ω_{xF} is the area of the load (from external load) plot M_{xF}; $\overline{M}_{xC}$ - ordinate taken from a single plot (from unit force) $\overline{M}_x$ below the center of gravity C of the load plot.

<u>Rule of signs.</u> If the multiplied plots lie on one side (both above or below), the product is positive; if the multiplied plots lie on different sides - the product is negative.

If the plot of the external load is piecewise linear in sections, and a single plot is always piecewise linear, the result of multiplication does not depend on the order of use of the coefficients, ie:

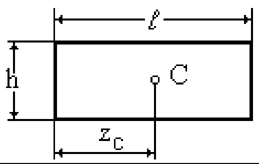
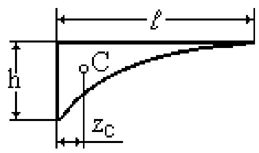
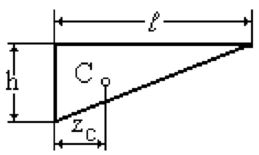
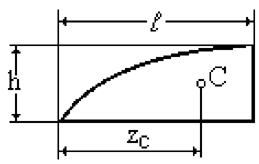
$$\frac{\omega_{xF} \cdot \overline{M}_{xC}}{EI_x} = \frac{\overline{\omega}_x \cdot M_{FC}}{EI_x}, \qquad (3.7)$$

where ω_{xF} is the area of the plot from the external load; $\overline{\omega}_x$ - plot area from a unit load; M_{FC} - ordinate under the center of gravity of a unit plot $\overline{M}_x$, taken from the plot M_{xF} from an external load.

If the plots M_{xF}, $\overline{M}_x$ consist of several sections, the multiplication is carried out by sections, and the result is summed, ie:

$$\Delta_{ij} = \sum_{i=1}^{n} \frac{\omega_{ixF} \cdot \overline{M}_{ixC}}{EI_x}. \qquad (3.8)$$

Note that in the considered problems the plots of load and unit bending moments consist of rather simple areas: rectangle, triangle, parabolic triangle, etc. The table shows the areas ω and the coordinates of the centers of gravity z_c of flat figures found in the plots.

	ω	z_C		ω	z_C
	$h\ell$	$\frac{1}{2}\ell$		$\frac{1}{3}h\ell$	$\frac{1}{4}\ell$
	$\frac{1}{2}h\ell$	$\frac{1}{3}\ell$		$\frac{2}{3}h\ell$	$\frac{5}{8}\ell$

When solving specific problems, it is advisable to use the trapezoidal rule for multiplying linear diagrams and the Simpson-Karnaukhov rule for multiplying any diagrams (in most cases nonlinear).

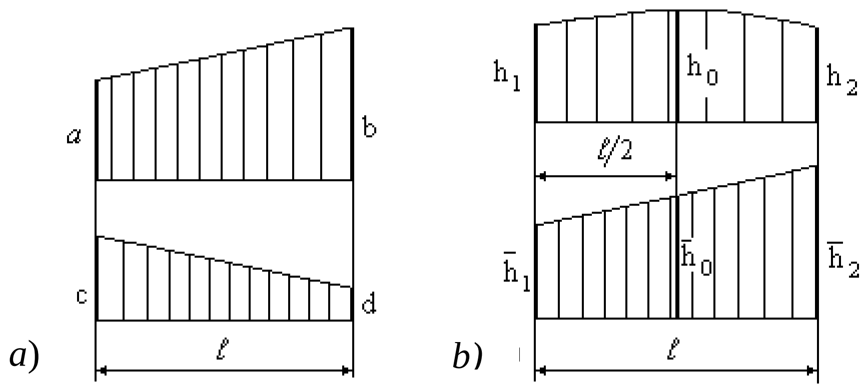


Figure 3.6 - Trapezoid and Simpson-Karnaukhov rule

Trapezoid rule (only for linear plots - Fig. 3.6, *a*). In the case when the plot M_{xF} (from the external load) is linear, the multiplication of the plots can be performed according to the trapezoidal rule. The result of multiplication of linear plots on a plot of length ℓ is equal to:

$$\Delta = \frac{\ell}{6EI_x}(a \cdot d + b \cdot c + 2 \cdot a \cdot c + 2 \cdot b \cdot d).$$

Rule of signs: if the ordinates that multiply are one sign (lie on one side), the resulting product is positive, if the signs of the ordinates are different - the product is negative.

Simpson-Karnaukhov rule (for linear diagrams and diagrams described by a square parabola, Fig. 3.6, *b*). The result of the product is as follows:

$$\Delta = \frac{\ell}{6EI_x}(h_1\bar{h}_1 + h_2\bar{h}_2 + 4h_0\bar{h}_0).$$

Here h_1, h_2 are the extreme ordinates of the load plot (nonlinear) on the site; $\bar{h}_1, \bar{h}_2$ - extreme ordinates of a unit plot (linear) on the site; h_0 and $\bar{h}_0$ - the average ordinates of the plots on the site. The rule of signs when multiplying ordinates is similar to the rule of trapezoid.

Example 3. For a cantilever beam loaded with external forces, as shown in Fig. 3.7, determine the deflection and angle of rotation at the end of the cantilever in the section A.

1. We define support reactions.

$\sum F_{yi} = 0$; $F - q \cdot 3a + R_B = 0 \Rightarrow R_B = 2qa$.

$\sum M_{Bi} = 0$; $-F \cdot 6a + q \cdot 3a \cdot 3{,}5a - M_B = 0 \Rightarrow M_B = 1{,}5qa^2$.

Check:

$\sum M_{Ai} = -q \cdot 3a \cdot 2{,}5a - M + R_B \cdot 6a - M_B = qa^2(-7{,}5 - 3 + 12 - 1{,}5) \equiv 0$.

2. We write down the expressions Q_y and M_x, build the corresponding plots.

$0 \le z_1 \le a$: $Q_y = F = qa$; $M_x = F \cdot z_1 = qaq \cdot z_1$.

$0 \le z_2 \le 3a$: $Q_y = F - q \cdot z_2 = qa - q \cdot z_2$.

Shear force changes sign at $z = a$.

$$M_x = F(z_2 + a) - q\frac{z_2^2}{2} = qa(z + a) - q\frac{z_2^2}{2}.$$

$$M_{x\,\text{max}} = M_x\big|_{z=\mathbf{a}} = 2qa^2 - 0{,}5qa^2 = 1{,}5qa^2;$$

$$M_x\big|_{z=1{,}5\mathbf{a}} = qa^2(2{,}5 - \frac{1{,}5^2}{2}) = 1{,}375qa^2.$$

$0 \le z_3 \le 2a$: $Q_y = -2qa$; $M_x = -M_B + R_B \cdot z = -1{,}5qa^2 + 2qa \cdot z$.

3. At the point A, we apply a unit force, build the plot $\overline{M}_x = -1 \cdot z$, defining the ordinates at the borders of each section.

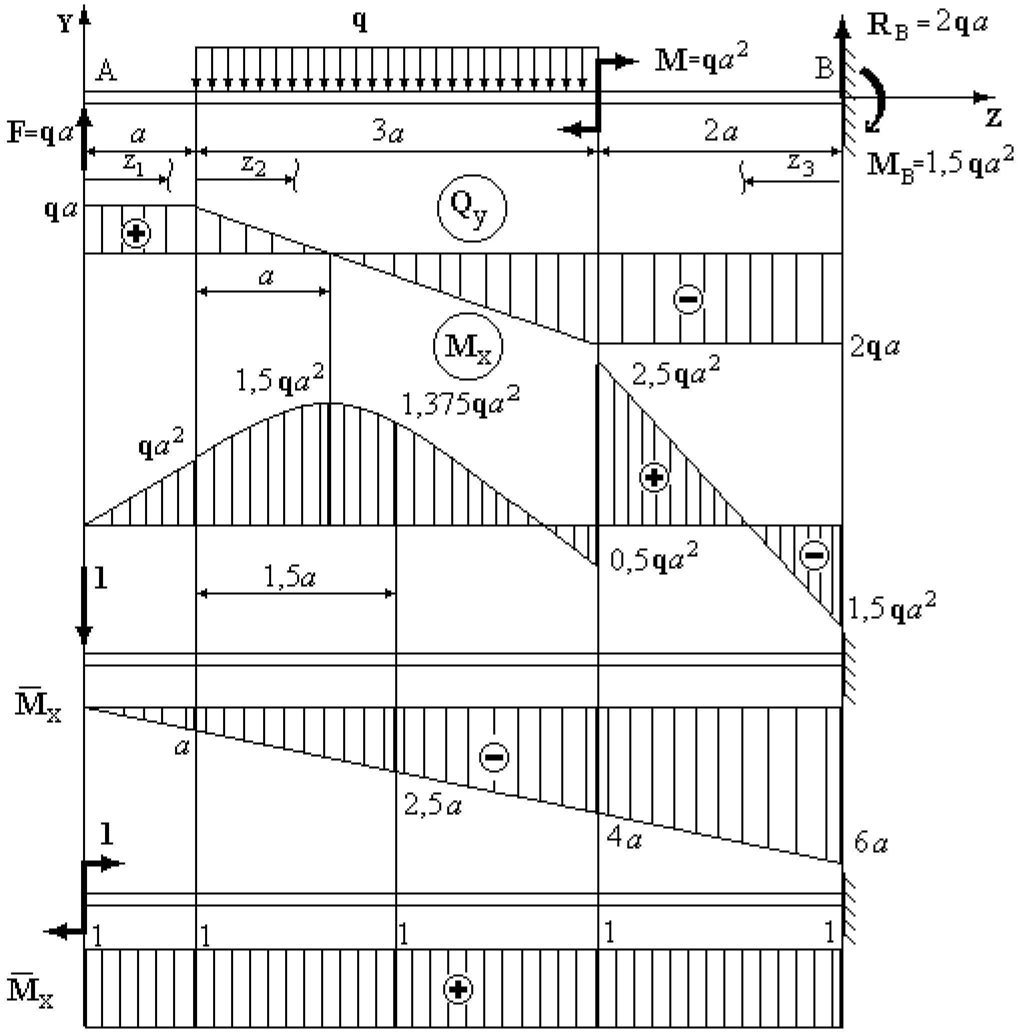


Figure 3.7 - Cantilever beam loaded with external forces

4. By multiplying the plots M_x and $\overline{M}_x$, we determine the desired deflection:

$$V_A = \frac{qa^4}{EI_x}[-\frac{1}{2}\cdot 1\cdot 1\cdot \frac{2}{3}\cdot 1+\frac{3}{6}(-1\cdot 1+4\cdot 0{,}5-4\cdot 1{,}375\cdot 2{,}5)+\frac{2}{6}(-2{,}5\cdot 6+$$

$$+4\cdot 1{,}5-2\cdot 2{,}5\cdot 4+2\cdot 1{,}5\cdot 6)]=\frac{qa^4}{6EI_x}[-2-38{,}25-22]=-\frac{10{,}375qa^4}{EI_x}.$$

Here, on the first section, the multiplication of epurs is performed according to Vereshchagin's rule; on the second - Simpson rule, on the third - trapezoid rule. The minus sign indicates that the section moves up under the action of an external load.

5. At the point A, we apply a unit moment, build the plot $\overline{M}_x = 1$. Multiplying it by sections with a load plot, we determine the angle of rotation of the section A.

$$\theta_A = \frac{qa^3}{EI_x}[\frac{1}{2}\cdot 1\cdot 1\cdot 1 + \frac{3}{6}(1\cdot 1 - 0{,}5\cdot 1 + 4\cdot 1{,}375\cdot 1) +$$

$$+\frac{2}{6}(2{,}5\cdot 1 - 1{,}5\cdot 1 + 2\cdot 2{,}5\cdot 1 - 2\cdot 1{,}5\cdot 1)] = \frac{qa^3}{EI_x}[0{,}5 + 3 + 1] = \frac{4{,}5qa^3}{EI_x}.$$

Here, on the first section, multiplication is performed according to Vereshchagin's rule, on the second - Simpson's, on the third - trapezoid. The result of the calculations is positive, therefore, the direction of the angle of rotation of the section A coincides with the direction of the unit moment.

4. GEOMETRIC CHARACTERISTICS OF CROSS SECTIONS

The resistance of the rod to various types of deformation depends not only on its material and dimensions, but also on the shape of the cross section and its orientation.

The main geometric characteristics of the object's cross-sections, which determine its resistance to various types of deformation, include cross-sectional areas, static moments, moments of inertia and moments of resistance. These characteristics for sections of a simple shape can be determined using special formulas, and for profiles of standard rolled steel (angles, channels, I-beams) - by tables of standards.

4.1. Static moments of section

Consider an arbitrary figure (cross-section of a beam), associated with the coordinate axes *Ox, Oy* (Fig. 4.1). Let's select an element of the area *dF* with coordinates *x, y*.

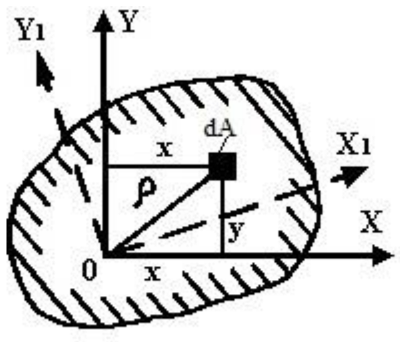


Figure 4.1 - Section of an arbitrary shape

By analogy to the expression of the moment of forces relative to any axis, expressions can also be formulated for the moment of area, which is called the *static moment of the section* and is defined as

$$S_x = \int_A y dA; \quad S_y = \int_A x dA \quad . \tag{4.1}$$

Static moments are expressed in units of cubic length (m³).

Let *Xc, Yc* be the coordinates of the center of gravity of the figure. Continuing the analogy with moments of forces, the following expression can be written on the basis of the theorem on the moment of uniform action:

$$S_x = Ay_C; \quad S_y = Ax_C,$$

where *A* is the area of the figure. Then the *coordinates of the center of gravity*

$$x_C = \frac{S_y}{A}; \quad y_C = \frac{S_x}{A}; \tag{4.2}$$

The static moments of the area relative to the central axes (axes passing through the center of gravity) are zero. To calculate the static moments of a complex figure, it is divided into simple parts, for each of which the area and position of the center of gravity are known. After that, the static moment of the area of the entire figure relative to the given axis is determined as the sum of the static moments of each part.

4.2. Area moment of inertia

The axial (equatorial) moment of inertia of a section is called the integral of products of elementary areas by the squares of their distances to the considered axes,

$$I_x = \int_A y^2 dA; \quad I_y = \int_A x^2 dA; \tag{4.3}$$

The polar moment of inertia relative to a given point (pole 0) is called the integral of products of elementary areas by the squares of their distances from the pole,

$$I_\rho = \int_A \rho^2 dA \quad , \tag{4.4}$$

The centrifugal moment of inertia of the section is called the integral of the products of the planes of the elementary plots at their distance from the coordinate axes,

$$I_{xy} = \int_A xydA \quad . \tag{4.5}$$

Moments of inertia are measured in units of length in the fourth degree (m^4).

Moments of inertia I_x, I_y, I_ρ are positive values and are related to each other by a simple relationship that follows from the equation $\rho^2 = x^2 + y^2$ (Fig. 4.1): $I_\rho = I_x + I_y$. From this dependence follows the property of invariance (constancy) of the sum $I_x + I_y$ when the coordinate axes are rotated. Since when rotating the axes X_1, Y_1 (Fig. 4.1) the value ρ for each area dA does not change, then

$$I_x + I_y = I_{x1} + I_{y1} \quad . \tag{4.6}$$

Note that the moment of inertia of a complex section is equal to the algebraic sum of the moments of inertia of its parts.

The value I_{xy} can be positive or negative depending on the location of the *X*, *Y* axes. A simple and practically important example when $I_{xy} = 0$, is the case of a symmetric section. Let, for example, the *Y* axis coincide with the axis of symmetry of the section (Fig. 4.2).

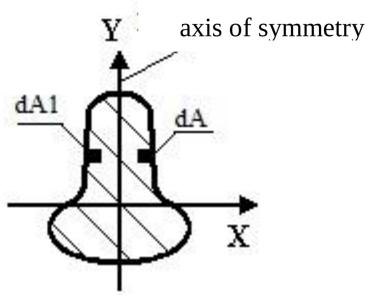


Figure 4.2 – Symmetrical section

Then, for each area dA with positive XY, there will be a symmetrical area dA_1 with the same size, but already negative XY. In sum, the centrifugal moment of inertia of these two platforms, and therefore I_{xy} of the entire section, will be zero.

4.3. Moments of inertia relative to parallel axes (Parallel axis theorem)

Often, when solving practical problems, it is necessary to determine the moments of inertia of a section relative to axes oriented in different ways in its plane. Therefore, it is important to establish dependencies between the moments of inertia of the same section relative to different axes.

Let the moments of inertia relative to the central axes X,Y be known, and it is necessary to determine the moments of inertia relative to the axes that are parallel to the central axes. (Fig. 4. 3).

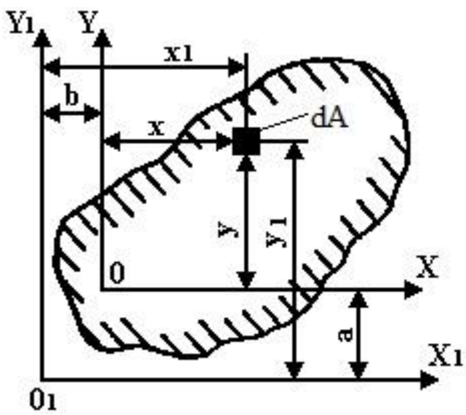


Figure 4.3 - Parallel transfer of coordinate axes

The distances between the axes are equal to *a* and *b*. Coordinates x_1, y_1 and x, y are related by the following dependencies: $x_1 = x + \boldsymbol{b}$; $y_1 = y + \boldsymbol{a}$.

Substituting these expressions into the integrals of the moments of inertia, after performing the appropriate transformations, we obtain the transition formulas for parallel axes:

$$I_{x1} = I_x + a^2 A \ ,$$
$$I_{y1} = I_y + b^2 A \ , \qquad (4.7)$$
$$I_{x1y1} = I_{xy} + ab\ A \ .$$

Note that the coordinates *a, b*, which are included in the last of these formulas, should be substituted taking into account their signs. Formulas (4.7) show that of all the moments of inertia relative to a series of parallel axes, the central moments of inertia will be the smallest.

4.4. Moments of inertia when turning axes

Let the moments of inertia of an arbitrary figure relative to the coordinate axes *X,Y* be known. Let's turn the *X,Y* axes to the angle α counterclockwise, considering the angle of rotation of the axes in this direction (Fig. 4.4).

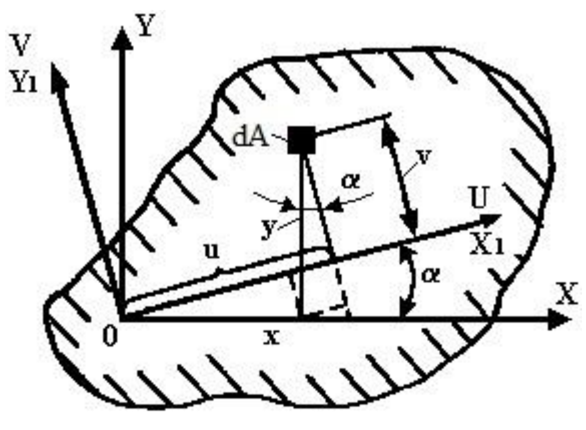


Figure 4.4 - Rotation of coordinate axes

Having expressed the coordinates of an arbitrary elementary platform in new axes through the coordinates of the previous system, after the appropriate transformations we obtain the transition formulas when the axes are rotated:

$$
\begin{aligned}
I_{x1} &= I_x \cos^2 \alpha + I_y \sin^2 \alpha - I_{xy} \sin 2\alpha \\
I_{y1} &= I_x \sin^2 \alpha + I_y \cos^2 \alpha + I_{xy} \sin 2\alpha \\
I_{x1y1} &= I_{xy} \cos 2\alpha - \frac{1}{2}\left(I_y - I_x\right) \sin 2\alpha \quad .
\end{aligned}
\tag{4.8}
$$

Note that these formulas obtained by rotating any system of mutually perpendicular axes are naturally also valid for the central axes.

4.5. The principal moments of inertia

Formulas (4.8) make it possible to establish how the moments of inertia of the section change when the axes are rotated by an arbitrary angle α.

For certain values of the angle α , the axial moments of inertia reach a maximum and a minimum. The extreme values of the axial moments of inertia are called the principal moments of inertia.

The axes relative to which the axial moments of inertia have extreme values are called the principal axes of inertia. The main axes of inertia are mutually perpendicular. The main central axes, the location of which is determined from

$$
\operatorname{tg} 2\alpha_0 = \frac{2I_{xy}}{I_y - I_x} \quad . \tag{4.9}
$$

The two values of the angle α_0 obtained from formula (4.9) differ from each other by 90° and give the position of the main axes, one of which is the axis of maximum (relative to it the axial moment of inertia of the section is maximum), and the second is the axis of minimum (relative to it the axial moment of inertia of the section is minimal).

The main moments of inertia I_{max}, I_{min} are determined by the formula , which can be easily obtained from dependencies (4.8) and (4.9):

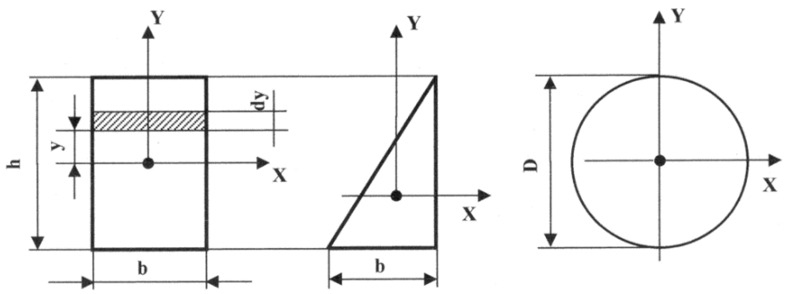
$$I_{\min}^{\max} = \frac{I_y + I_x}{2} \pm \sqrt{\left(I_y - I_x\right)^2 + 4I_{xy}^2} \; . \qquad (4.10)$$

4.6. Moments of inertia for some sections

Consider simple cross-sections in the form of a rectangle and a triangle with bases *b* and height *h* and a circle with diameter *D*. We need to find their moments of inertia relative to the central axis *X*.

Solution.

According to the definition of the axial moment for a rectangle



$$I_x = \frac{bh^3}{12} \qquad I_x = \frac{bh^3}{36} \qquad I_x = \frac{\pi D^4}{64}; \quad I_p = \frac{\pi D^4}{32}$$

Figure 4.5– Common cross-sections

$$I_x = \int y^2 dA = b \int_{-\frac{h}{2}}^{\frac{h}{2}} y^2 dy = \frac{bh^3}{12} \; . \qquad (4.11)$$

The formula for the axial moment of inertia of a triangle 4.12 (Figure 4.5, b) and the formula for the axial moment of inertia of a circle 4.13 are derived similarly.

$$I_x = \frac{bh^3}{36} \tag{4.12}$$

$$I_x = \frac{\pi D^4}{64} \tag{4.13}$$

The polar moment of inertia of the circle is determined

$$I_p = \int \rho^2 dA = 2\pi \int_0^{\frac{D}{2}} \rho^3 d\rho = \frac{\pi D^4}{32}\ . \tag{4.14}$$

It should be noted that the last value is two times greater than the axial moment of inertia of the circle. This also follows from the equation $I_p = I_x + I_y = 2I_x$, since for a circle $I_x = I_y$.

The radii of inertia of the figure are called quantities

$$i_x = \sqrt{\frac{I_x}{A}}, \quad i_y = \sqrt{\frac{I_y}{A}}\ . \tag{4.15}$$

4.7. Resistance Moment (section modulus)

The resistance moments (section modulus) of the figure are the ratio of the corresponding moment of inertia to the distance to the farthest point.

Axial

$$W_x = \frac{I_x}{Y_{\max}}\ , \qquad W_y = \frac{I_y}{X_{\max}}\ . \tag{4.16}$$

Polar

$$W_p = \frac{I_p}{\rho_{\max}}\ . \tag{4.17}$$

5. GENERAL INSTRUCTIONS FOR PERFORMING CALCULATION AND DESIGN WORKS AND THE REQUIREMENTS FOR THEIR FORMATION

Resistance of materials occupies the most important place among the fundamental disciplines in the cycle of general engineering training of students.

The goal of the course is for future engineers to acquire practical skills in strength calculations.

When starting work, the student must:

study the theoretical material of the corresponding section of the course according to the synopsis and additional educational literature;

work through examples and problems that were solved at lectures and practical classes;

analyze the initial data and the statement of the problem and outline a plan for its implementation;

make initial presentations: draw the diagram of the beam, write down the formulas and equations necessary for solving the problem (equation of static equilibrium, etc.).

draft the task and, if necessary, consult the teacher.

Calculations must be made in a general form with intermediate statements in ordinary or decimal fractions, keeping three significant figures everywhere. To build the graphs of internal force factors, stresses and displacements, it is necessary to correctly choose the scale along the coordinate axes, mark the appropriate parameter and its dimension, and then draw a graph based on the required number of points.

The assignment is subject to credit if the following conditions are met:

the finished version of the calculation and design task was passed and the correct answers to the control questions were given;

the control tasks at the consultations were solved.

The final version of the task is drawn up on sheets of A4 paper.

Output data:

- load diagrams of five beams /Table 5.1/;
- magnitudes of external forces and geometric dimensions of the beams, strength characteristics of beam material (yield strength for steel and strength limit for cast iron), safety factor /Table 5.2/;
- configuration of the complex cross-section of the beam /Table 5.3/.

Task execution procedure.

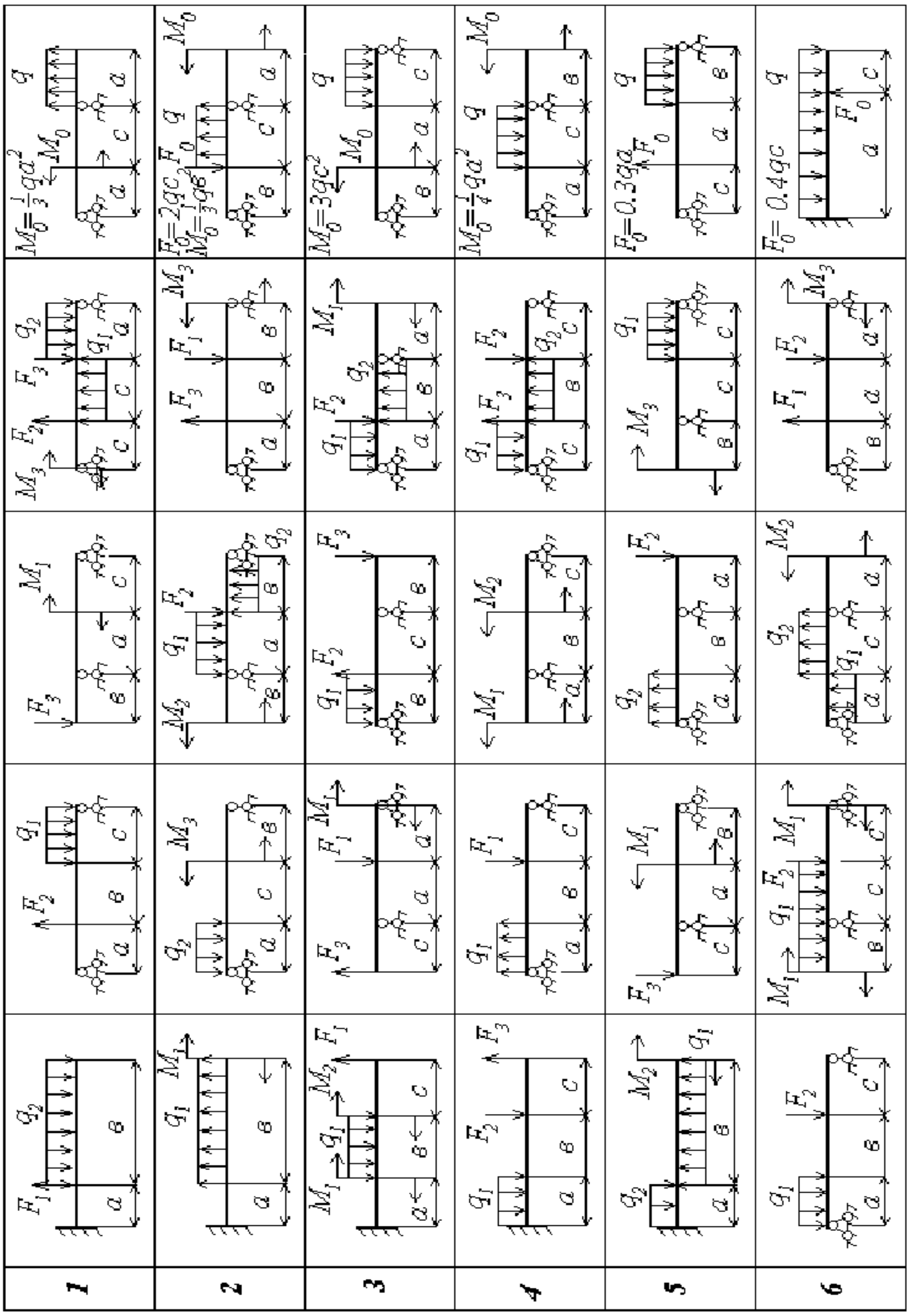
1. Construct diagrams of internal transverse forces and bending moments for all beams.
2. Select a number of simple sections of the beam marked by the teacher according to the condition of strength under normal stresses and make a comparative analysis of the degree of their rationality.
3. Determine the margin of strength n_T^0 of the same beam, if its cross-section has a complex configuration /Table 5.3/.
4. Determine the permissible value of external forces for one example of a beam from the table 1.

Solution plan

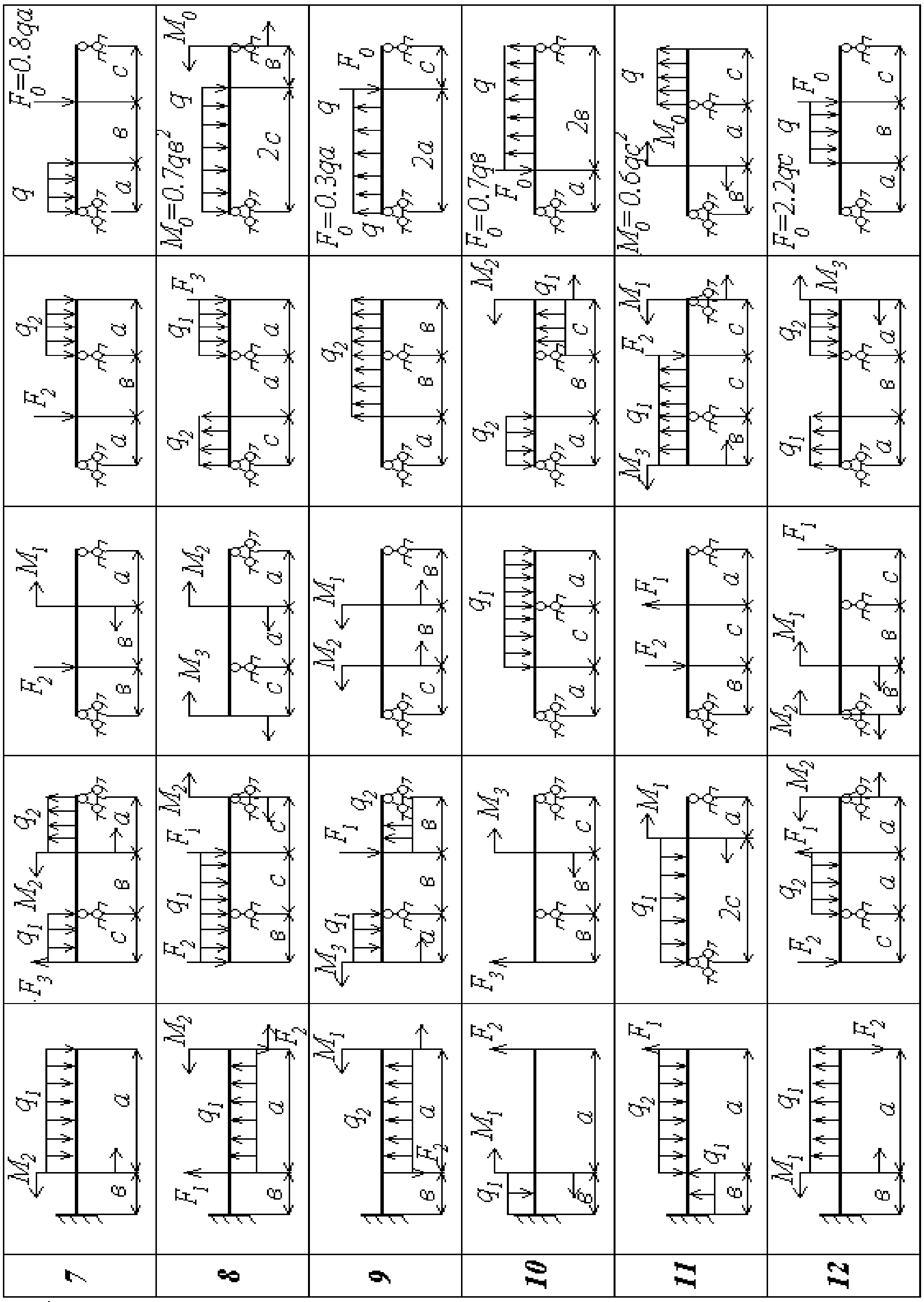
1) According to the given option, draw the calculation schemes of the beams together with their load, observing a certain scale.
2) For each beam, determine the support reactions (numerically or in general form), make expressions and calculate the internal forces Q_y , M_x in all areas, draw their diagrams and check the correspondence of the diagrams to the differential ratios between the force factors during bending.
3) For each beam, find the dangerous section from the point of view of strength under normal stresses and determine the maximum bending moment $M_{X\max}$ by module.
4) Calculate the permissible stress.

5) Determine the necessary axial moment of resistance W_X from the solution of the strength condition.
6) Select simple sections in the form:
   – a rectangle with aspect ratios $h/b = 2$;
   – a rectangle with the inverse aspect ratio $b/h = 2$;
   – a circle;
   – a ring in which $\alpha = d/D = 0.8$;
   – I-beam.
   Calculate the cross-sectional area. Submit the obtained results in the form of a table (see the appendix).
7) For dangerous cross-sections of the I-beam, construct normal and tangential stress distribution graphs with calculation of their values at characteristic points of the cross-section.
8) In the section of the I-beam beam specified by the teacher, find its deflection.
9) For a beam of complex cross-section, calculate the margin of strength n_T^0 relative to the yield point. For this you need:
   – determine the coordinates X_C, Y_C of the center of gravity of the cross-sectional area, as well as the position of the main central axes;
   – find the neutral axis and the distance from it to the farthest cross-section point $Y_{\max}$;
   – calculate the axial moment of inertia I_X relative to the neutral axis X;
   – find the largest stresses $\sigma_{\max}$ in the cross-section of the beam and construct the graph σ.
   – calculate the margin of safety n_T^0.
10) Select the allowable value of external forces for the generally given beam scheme according to the condition of strength under normal stresses. Accept the material of the beam - cast iron KCh 30-6 with different values of yield strength in tension-compression and a complex cross-section of its version /Table 3/.

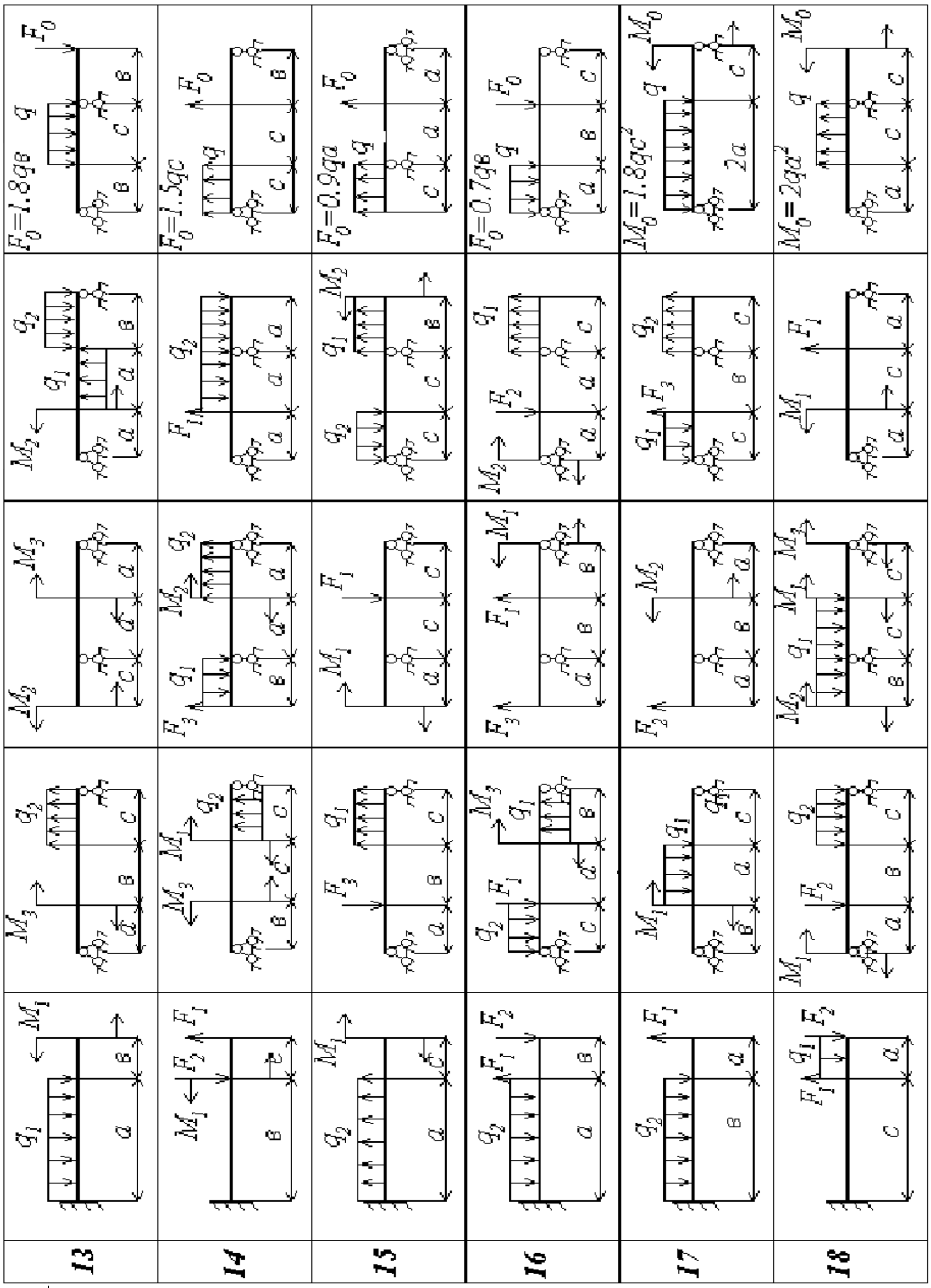
Table 5.1 - Load diagrams of five beams



Continuation of the Table 5.1



Continuation of the
Table 5.1



Continuation of the Table 5.1

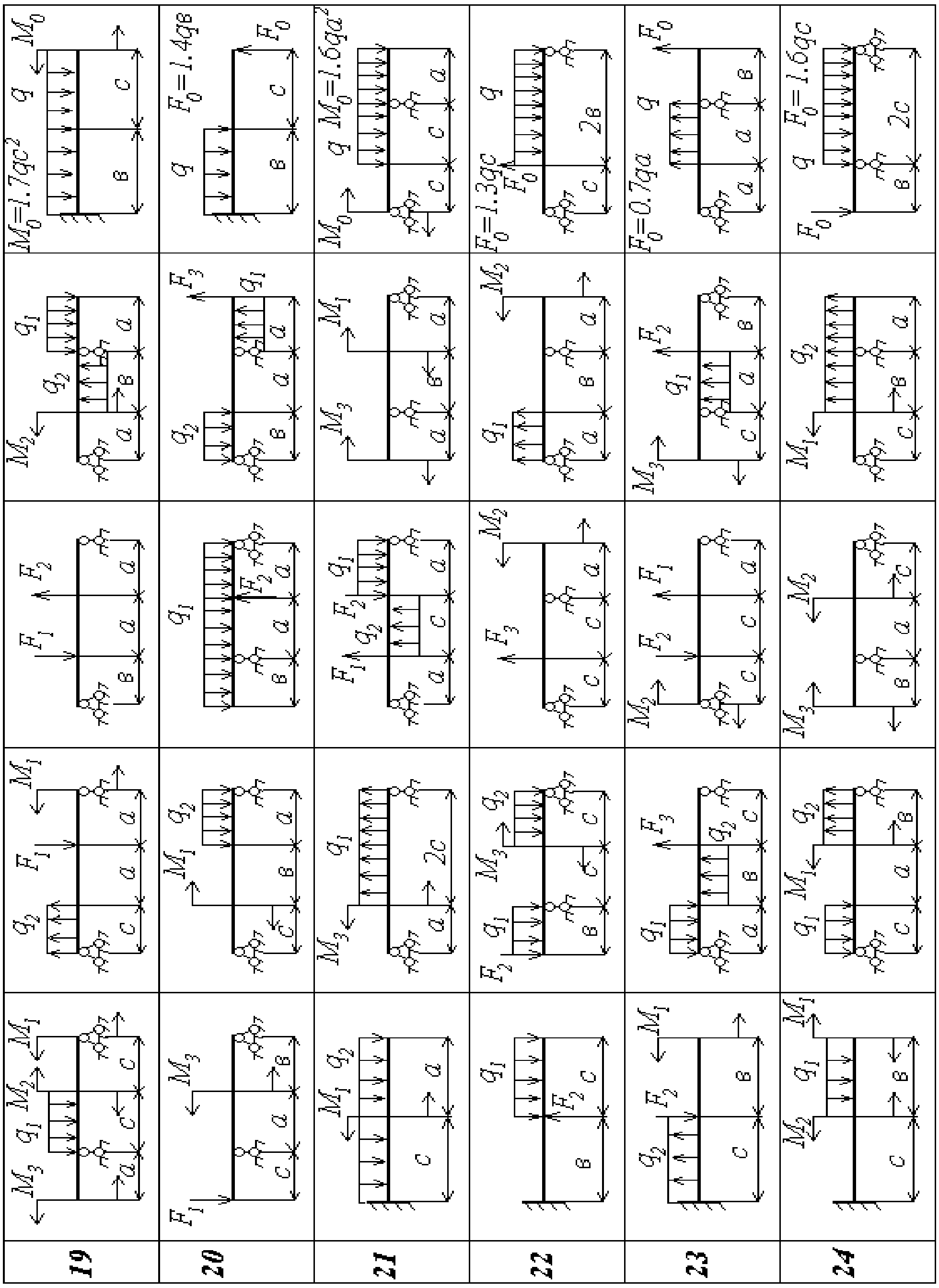


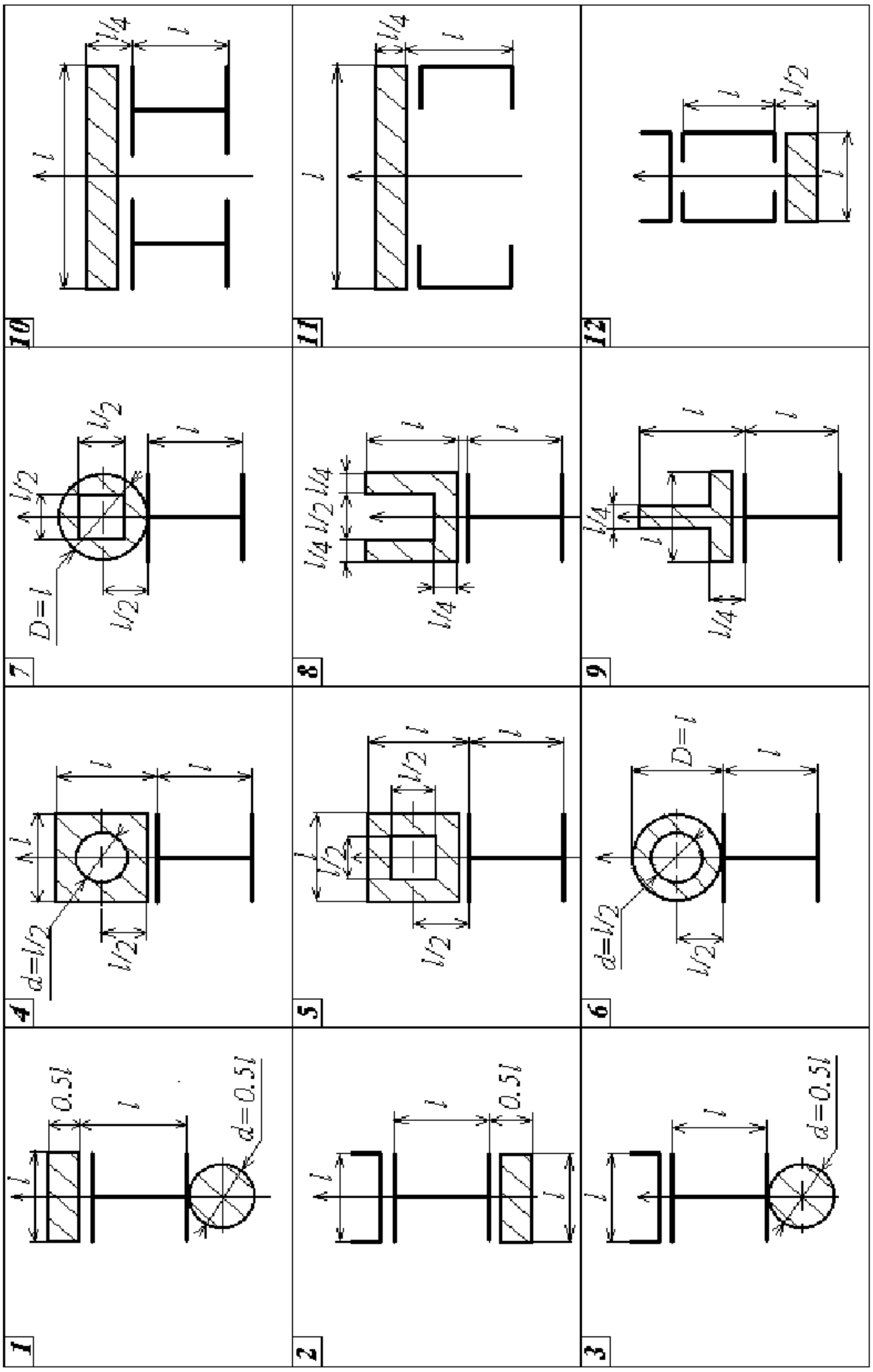
Table 5.2 - Output data

Variant	F_1	F_2	F_3	M_1	M_2	M_3	q_1	q_2	n_T	a	b	c	L
	kN			kN			kN/m	kN/m		m			cm
1	10	20	30	40	20	10	30	10	1,7	1,3	1,2	2,1	12
2	30	10	20	30	30	20	50	30	1,4	2,2	2,5	1,8	14
3	20	20	10	20	40	20	40	10	1,5	1,3	1,6	2,1	16
4	10	30	20	20	10	20	20	40	1,6	2,3	2,6	1,5	18
5	20	30	10	30	40	20	20	30	1,8	1,7	1,8	2,1	20
6	30	10	20	10	30	40	10	20	2,0	1,4	2,1	2,5	22
7	30	20	10	30	40	10	20	10	1,8	2,4	2,9	1,8	20
8	50	10	20	20	20	30	30	20	1,6	1,7	2,1	1,8	18
9	30	20	10	20	10	30	10	40	1,5	1,1	1,7	2,5	16
10	10	40	30	10	40	20	20	30	1,4	1,3	2,6	1,8	14
11	20	20	20	30	10	10	30	10	1,6	1,4	1,9	2,3	12
12	20	30	40	20	30	40	40	30	1,4	2,4	2,9	1,8	10
13	40	10	20	10	20	40	50	20	1,5	1,1	2,5	2,7	14
14	50	20	10	20	10	20	10	30	1,6	1,7	1,2	2,5	16
15	30	10	30	30	30	10	10	40	1,8	2,7	2,1	1,7	18

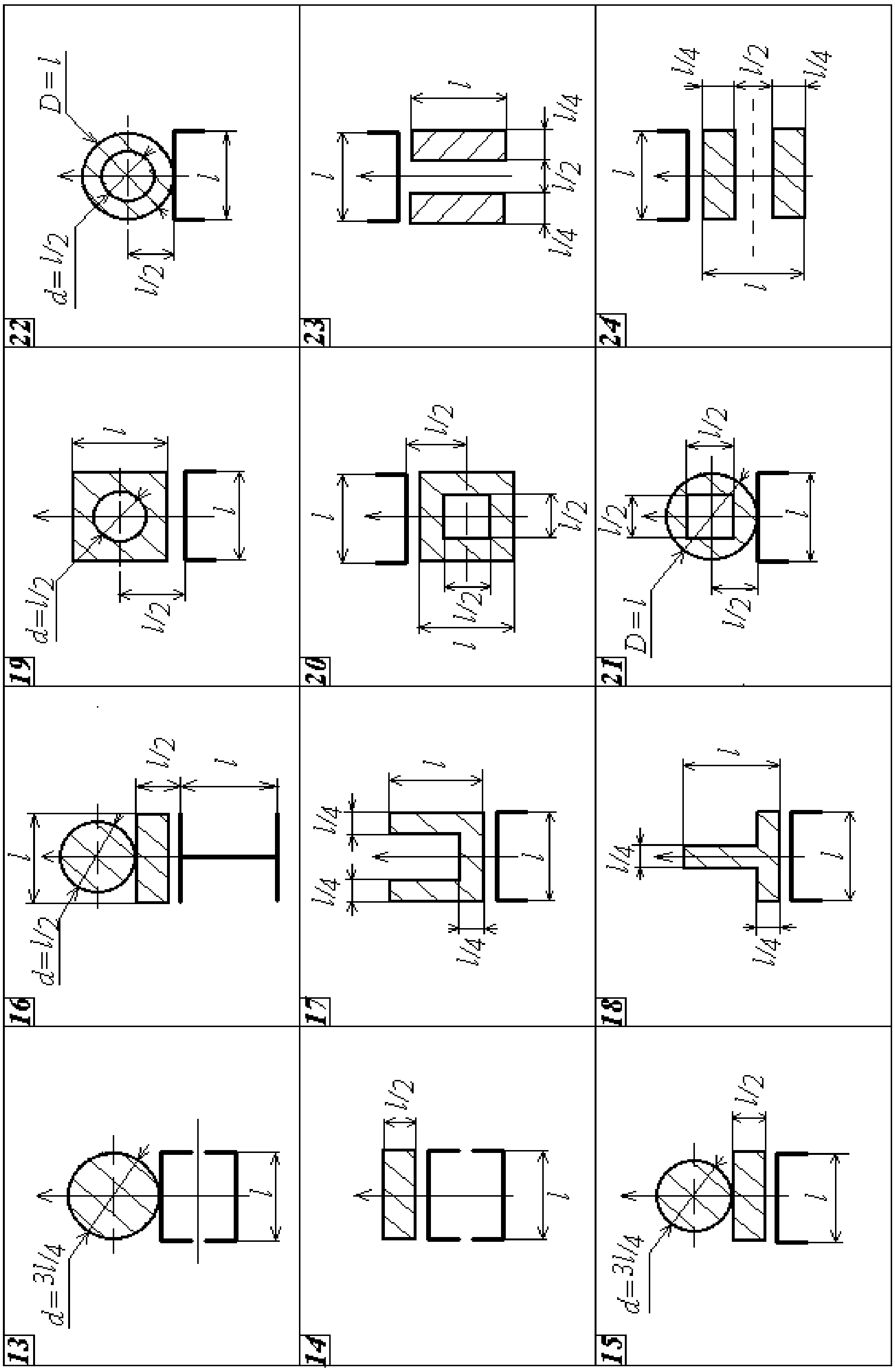
Steel : $\sigma_T = 300$ MPa;

Cast iron KCh30 – 6 : $\sigma_T^{+} = 190$ MPa, $\sigma_T^{-} = 210$ MPa .

Table 5.3 -
Complex
cross-section
of the beam



Continuation of the
Table 5.3



Task performance sequence

First, it is necessary to build the graphs of the internal force factors during bending for all the beams of the variant of the task. The strength condition under normal stresses must be fulfilled at the most dangerous point of the dangerous section:

$$\sigma = \frac{M_{X\max}}{I_X} Y_{\max} \leq [\sigma] \ . \tag{5.1}$$

Or for sections symmetrical about the neutral line:

$$\sigma = \frac{M_{X\max}}{W_X} \leq [\sigma] \ . \tag{5.2}$$

The section where the maximum in absolute value bending moment $M_{X\max}$ acts on the M_X plot is considered dangerous.

The dangerous point in the section has the $Y_{\max}$ coordinate and is located at the greatest distance from the neutral line - the X axis.

The permissible stress is determined by the formula:

$$[\sigma] = \frac{\sigma_T}{n_T} \ . \tag{5.3}$$

The necessary moment of resistance is found from the condition of strength

$$W_X \geq \frac{M_{X\max}}{[\sigma]} \ . \tag{5.4}$$

The selection of the necessary dimensions of the cross-sections of the beam is carried out as follows:

- for a I-beam, the profile number is determined by comparing the found moment of resistance with the assortment data;
- for a rectangle, the axial moment of resistance is determined by the dimensions of the sides h and b as:

$$W_X = \frac{bh^2}{6} ,$$

where *h* is the side of the rectangle perpendicular to the *X* axis.

Because

$$h/b = k , \qquad / \ k = 2;\ 0,5 \ /,$$

then $b = h/k$ and $W_X = h^3/6k$ from where $h = \sqrt[3]{6kW_X}$,

Cross-sectional area $A = b \times h$

- for a circle $W_X = \frac{\pi d^3}{32} \approx 0.1d^3$, then $d = \sqrt[3]{10W_X}$ and the cross-sectional area is $A = \pi d^2/4$.
- for the ring $W_X = \frac{\pi D^3}{32}\left(1-\alpha^4\right) \approx 0.1D^3\left(1-\alpha^4\right)$, where $\alpha = d/D$; *d*, *D* are the inner and outer diameters of the ring, respectively. Area of the ring $A = \frac{\pi D^2}{4}\left(1-\alpha^4\right)$.

Determination of the safety margin of a complex section:

1. According to the parameter *L* (Table 5.2) and the assortment of standard profiles, set the characteristic dimensions of the section and depict it, observing the scale.
2. Break the cross-section into such simple component parts, the center of gravity of which and the axial moments of inertia are known or easily found.
3. For each constituent part of the section, determine and draw its own main central axes X_i, Y_i.
4. Calculate the coordinates of the center of gravity of the complex section:

$$X_C = \frac{\sum_{i=1}^{N} S_{Yi}}{\sum_{i=1}^{N} A_i} = \frac{\sum_{i=1}^{N} A_i X_{Ci}}{\sum_{i=1}^{N} A_i}, \quad Y_i = \frac{\sum_{i=1}^{N} S_{Xi}}{\sum_{i=1}^{N} A_i} = \frac{\sum_{i=1}^{N} A_i Y_{Ci}}{\sum_{i=1}^{N} A_i}, \tag{5.5}$$

where S_{Yi}, S_{Xi} – static moments of the *i*-th component of the cross-section relative to any fixed system of axes X_0, Y_0; X_{Ci}, Y_{Ci} – coordinates of the centers of gravity of the *i*-th component of the cross-section in the selected coordinate system X_0, Y_0; A_i is the area of the ith component of the section. The summation in (5.5) is carried out by the number *N* of constituent parts of the section.

5. Draw the main central axes *X*, *Y* of the complex section.
6. Determine the axial moment of inertia I_X of the given section, taking into account the following:

if the main central axes X_i, Y_i of the constituent part of the section are parallel to the main central axes *X*, *Y*, then

$$(I_X)_i = I_{Xi} + a_i^2 A_i , \tag{5.6}$$

where

a_i – is the distance between the *X* and X_i axes;

I_{Xi} – the moment of inertia of the component part of the section relative to its own axis X_i ; the moment of inertia of a complex figure relative to the main central axis is equal to the sum of the moments of inertia of its component parts relative to the same axis:

$$I_X = \sum_{i=1}^{N} I_{Xi} . \tag{5.7}$$

So,

$$I_X = \sum_{i=1}^{N} \left(I_{Xi} + a_i^2 A_i \right) . \tag{5.8}$$

Find the maximum stresses using formula (5.1).

7. Calculate the margin of strength of a complex section according to (5.9)

$$n_T = \sigma_T / \sigma_{\max} \quad . \tag{5.9}$$

It is sufficient to construct normal and tangential stress distribution graphs only for a beam of I-beam section. Determine the contours of normal stresses σ in the section where $M_{X\max}$ acts, and find the distribution of tangential stresses τ for the section with the maximum by module shear force $Q_{Y\max}$. On the plot σ, mark the zones of tension and compression, and on the plot - the direction of the vector of tangential stresses. Calculation of tangential stresses τ should be carried out according to the formula of D. I. Zhuravskyi (5.10) for characteristic points of the section:

– the furthest from the neutral axis;

– that lie at the junction of the I-beam shelves with the wall;

– lying on the neutral axis of the section:

$$\tau = \frac{Q_{y\max} \cdot S_x^{cut}}{b \cdot I_x}, \tag{5.10}$$

here $Q_{Y\max}$ is the maximum internal force in the beam by modulus;

S_X^{cut} – static moment of the cut-off part of the cross-sectional area at the level where the tangential stress relative to the neutral axis X is determined;

I_X – axial moment of inertia of the section;

b – is the width of the cross-sectional area at the level where the tangential stress is determined.

8. Movements (deflection) in beams are found according to the energy method using Mohr's integral:

$$V = \int_l \frac{M_X \cdot \overline{M}_i}{EI_X} dz \ , \tag{5.11}$$

which can be calculated by Vereshchagin's rule.

$$V = \frac{\left[M_X \cdot \overline{M}_i \right]}{EI_i} \quad . \tag{5.12}$$

In relation (5.12) M_X – plot of bending moment from external forces;

$\overline{M}_i$ – the plot of the «fictitious» bending moment from the unit force applied in the section where the deflection is located.

It should be taken into account that the expression (5.12) can be calculated by a graphoanalytical method according to the formulas given in Fig. 5.1, while the general curves of bending moments $M_X, \overline{M}_i$ must be divided into parts within which the specified functions remain unchanged.

9. Choose the permissible value of the external force according to the strength conditions for a complex cross-section, taking into account the different tensile and compressive properties of cast iron.

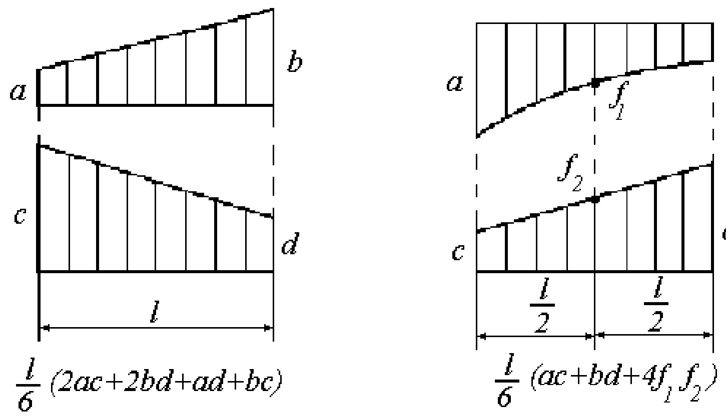


Figure 5.1 - Graphoanalytical method

Appendices

Appendix 1 - A sample task

A sample task

Initial data: n_T = 1.5, L = 10 cm

The material of beams №1- 4 is steel

yield stress σ_T = 300 MPa

$E = 2 \cdot 10^5$ MPa

The material of beam №5 is cast iron

КЧ 30 – 6

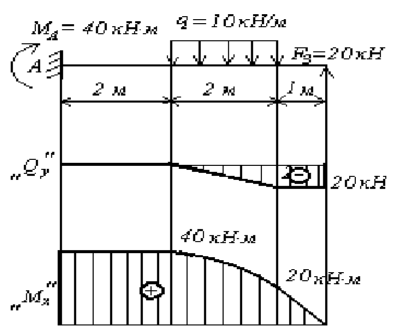
ultimate stress

$$\sigma^{+} = 190 \text{MPa}$$

$$\sigma^{-} = 210 \text{MPa}$$

E=2,0 10^5 MPa

1



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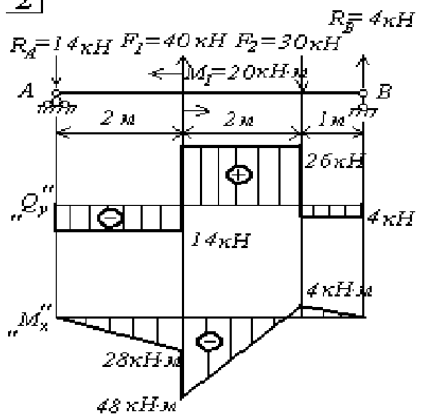

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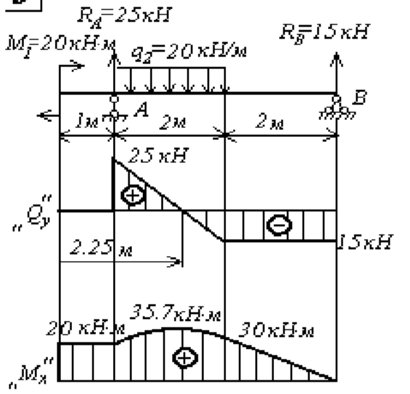

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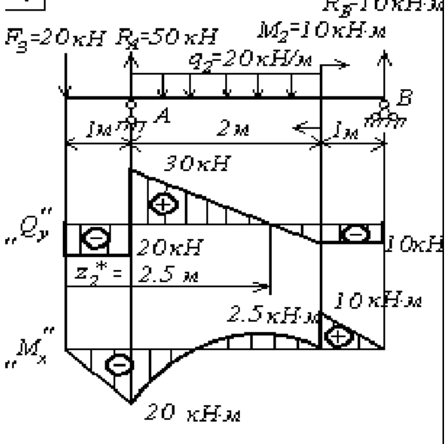

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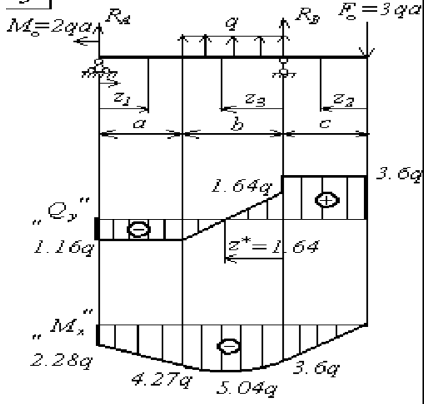


Calculation of beam №4:

1. Determination of reactions

$\sum M_A = 0;\ \ F_3 \cdot 1 - q_2 \cdot 2 \cdot 1 - M_2 + R_B \cdot 3 = 0;\ \ R_B = (-20 \cdot 1 + 20 \cdot 2 \cdot 1 + 10)/3 = 10$ кN;
$\sum M_B = 0;\ \ F_3 \cdot 4 - R_A \cdot 3 + q_2 \cdot 2 \cdot 2 - M_2 = 0;\ \ R_A = (20 \cdot 4 + 20 \cdot 2 \cdot 2 - 10)/3 = 50$

Check: $\sum F_{iy} = 0;\ \ R_A + R_B - F_3 - q_2 \cdot 2 = 0;\ \ 50 + 10 - 20 - 20 \cdot 2 = 0$.

2. Determination of shear forces Q_Y and bending moments M_X

$0 \le z_1 \le 1m \qquad Q_y(z_1) = -F_3 = -20\,\text{kN};\ \ M_x(z_1) = -F_3 z_1;$

$M_{x|z_1=0} = 0;\ \ M_{x|z_1=1м} = -20\,\text{kNm};$

$1m \le z_2 \le 3m \qquad Q_y(z_2) = -F_3 + R_A - q_2(z_2 - 1);$

$Q_{y|z_2=1м} = -20 + 50 = 30\,\text{kN};\qquad Q_{y|z_2=3м} = -20 + 50 - 20 \cdot 2 = -10\,\text{kN};$

$$M_x(z_2) = -F_3 z_2 + R_A(z_2 - 1) - \frac{q_2(z_2-1)^2}{2};$$

$$M_{x|z_2=1м} = -20 \cdot 1 = -20\,kNm;\quad M_{x|z_2=3м} = -20 \cdot 3 + 50 \cdot 2 - 20\frac{2^2}{2} = 0;$$

$$M_x'(z_2) = Q_y(z_2) = -F_3 + R_A - q_2(z_2^* - 1) = 0;$$

$$z_2^* = (-F_3 + R_A + q_2 \cdot 1)/q_2 = 2.5\,м;$$

$$M_{x|z_2=2.5м} = -20 \cdot 2.5 + 50 \cdot 1.5 - 20\frac{1.5^2}{2} = 2.5\text{kNm};$$

$0 \le z_3 \le 1m \qquad Q_y(z_3) = -R_B = -10\,\text{kN};\ \ M_x(z_3) = R_B \cdot z_3;$

$M_{x|z_3=0} = 0;\ \ M_{x|z_3=1м} = 10\,\text{kNm}$

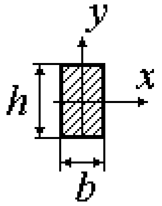
3. Selection of sections based on strength conditions

$$\sigma_{\max} = \frac{M_{X\max}}{W_X} \le [\sigma];\ \ W_X \ge \frac{M_{X\max}}{[\sigma]};\quad [\sigma] = \frac{\sigma_T}{n_T} = \frac{300}{1.5} = 200\ \text{MPa};$$

$$W_X \ge \frac{20 \cdot 10^3 \cdot 10^6}{200 \cdot 10^6} = 100\,\text{cm}^3.$$

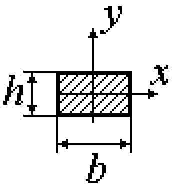
We accept I №16: $I_x = 873\ \text{cm}^4$, $W_x = 109\ \text{cm}^3$, $A = 20.2\ \text{cm}^2$,

$S_{x\max} = 62.3\,\text{cm}^3$, $h = 16$ cm, $d = 5$ mm, $t = 7.8$ mm, $b = 81$ mm .



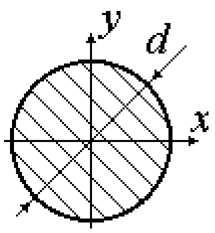
$$\frac{h}{b} = 2, \qquad Wx = \frac{\frac{h}{2} \cdot h^2}{6} = 100\,\text{cm}^3,$$

$$h = \sqrt[3]{12 \cdot 100} = 10.6\,\text{cm}, \quad b = 5.3\,\text{cm}.$$



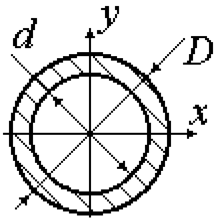
$$\frac{h}{b} = \frac{1}{2}, \quad W_x = \frac{2h \cdot h^2}{6} = 100\,\text{cm}^3,$$

$$h = \sqrt[3]{3 \cdot 100} = 6.7\,\text{cm}, \quad b = 13.4\,\text{cm}.$$



$$W_x = \frac{\pi d^3}{32} \approx 0.1 d^3 = 100\,\text{ñm}^3,$$

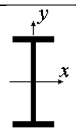
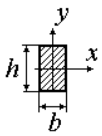
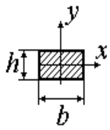
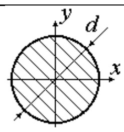
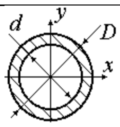
$$d = \sqrt[3]{10 \cdot 100} = 10\,\text{ñm}.$$



$$W_x = \frac{\pi \cdot D^3}{32}(1-\alpha^4) \approx 0.1 D^3 (1-\alpha^4) = 100\,\text{cm}^3,$$

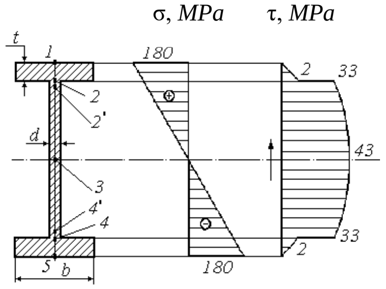
$$D = \sqrt[3]{\frac{10 \cdot 100}{(1-0.8^4)}} = 12\,\text{cm},$$

$$d = \alpha \cdot D = 0.8 \cdot 12 = 9.6\,\text{cm}.$$

						
A, cm^2	20.2	56	90	78	40	
A_i/A_I	1	2.8	4.5	3.9	2	

4. Stress in the I-beam

$$\sigma_{\max} = \frac{20 \cdot 10^3}{109 \cdot 10^{-6} \cdot 10^6} = 180\,\text{ÌPà} \quad ;$$



$$\tau_{1,5} = 0;$$

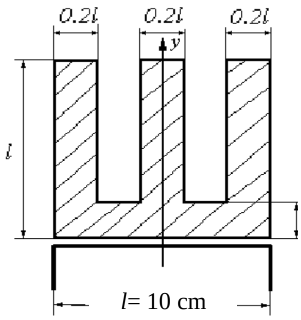
$$\tau_{2,4} = \frac{30 \cdot 10^3 \cdot 81 \cdot 10^{-3} \cdot 7.8 \cdot 10^{-3} \cdot 7.51 \cdot 10^{-2}}{81 \cdot 10^{-3} \cdot 873 \cdot 10^{-8} \cdot 10^6} = 2\,\text{ÌPà} \quad ;$$

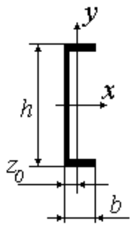
$$\tau_{2',4'} = \frac{30 \cdot 10^3 \cdot 81 \cdot 10^{-3} \cdot 7.8 \cdot 10^{-3} \cdot 7.51 \cdot 10^{-2}}{5 \cdot 10^{-3} \cdot 873 \cdot 10^{-8} \cdot 10^6} = 33\,\text{ÌPà} \quad ;$$

$$\tau_3 = \frac{30 \cdot 10^3 \cdot 62.3 \cdot 10^{-6}}{5 \cdot 10^{-3} \cdot 873 \cdot 10^{-8} \cdot 10^6} = 43\,\text{MPa}$$

5. Determination of the safety factor n_T^0 of a complex section

An extract from the assortment for channel №10 gives:



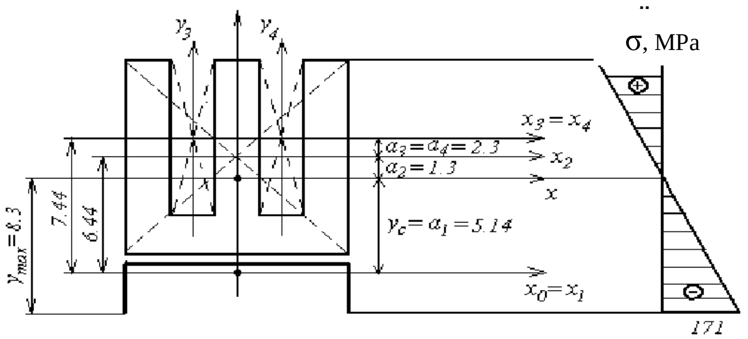


$I_x = 174\,\text{ñm}^2,\ I_y = 20.4\,\text{ñm}^4,\ A = 10.9\,\text{ñm}^2,$

$b = 46\,\text{mm},\quad z_0 = 1.44\,\text{ñm},\quad h = 10\,\text{ñm}.$

Then for a complex section we have:

$$x_c = 0; \qquad y_c = \frac{10^2 \cdot 6.44 - 2(2 \cdot 8 \cdot 7.44)}{10.9 + 10^2 - 2 \cdot 2 \cdot 8} = 5.14\,\text{ñm};$$



$$I_x = (20.4 + 5.14^2 \cdot 10.9) + (\frac{10^4}{12} + 1.3^2 \cdot 10^2) - 2(\frac{2 \cdot 8^3}{12} + 2.3^2 \cdot 16) = 971\,\text{ñm}^4;$$

$$y_{í.â.} = 8.3\,\text{ñm}; \quad \sigma_{max} = \frac{20 \cdot 10^3 \cdot 8.3 \cdot 10^{-2}}{971 \cdot 10^{-8} \cdot 10^6} = 171 \;\; \text{MPa},$$

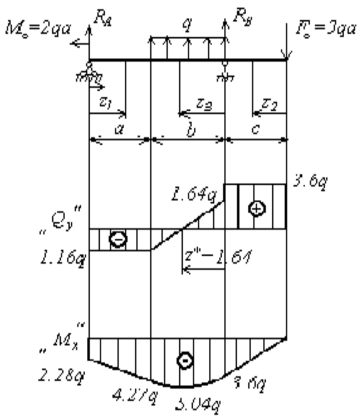
$$y_{â.â.} = 6.3\,\text{cm} \Rightarrow \sigma_{â.â.} = \frac{20 \cdot 10^3 \cdot 6.3 \cdot 10^{-2}}{971 \cdot 10^{-8} \cdot 10^6} = 129 \;\; \text{ÌPà}, \quad n_T{}^0 = \frac{\sigma_T}{\sigma_{max}} = \frac{300}{171} = 1{,}75;$$

6. Calculation of the permissible external force for a beam of complex cross-section (see clause 5)

The material is cast iron КЧ 30 – 6,

$\sigma_{\text{T}}^{+} = 190$ ÌPà, $\sigma_{\text{T}.}^{-} = 210$ ÌPà, $n_{\text{Ò}} = 1.5$, a =1.2 m, b =2.8 m, c =1 m .

Determination of reaction



$$\sum M_A = 0; \quad M_0 + qb(a + \frac{b}{2}) + R_B(a+b) - F_0(a+b+c) = 0;$$

$$R_B = \frac{F_0(a+b+c) - qb(a+\frac{b}{2}) - M_0}{a+b} = \frac{3qa(a+b+c) - qb(a+\frac{b}{2}) - 2qa^2}{a+b} = 1{,}96q;$$

$$\sum M_B = 0; \quad M_0 - R_A(a+b) - q\frac{b^2}{2} - F_0 c = 0;$$

Check: $\sum F_y = 0: \quad R_A + R_B - qb - F = q(-1.16 + 2.8 + 1.96 - 3 \cdot 1.2) \equiv 0.$

Determination of Q_Y and M_X

1) $0 \le z_1 \le a$

$Q_y = R_A = -1.16q;$

$$M_X = R_A z_1 - M_0 = \begin{cases} z_1 = 0; & -2.28q \\ z_1 = a; & -4.27q \end{cases}$$

2) $0 \le z_2 \le c$

$Q_y = F_0 = 3.6q;$

$$M_X = -F_0 z_2 = \begin{cases} z_2 = 0; & 0 \\ z_2 = c; & -3.6q \end{cases}$$

3) $0 \le z_3 \le b$

$$Q_y = F_0 - R_B - qz_3 = \begin{cases} z_3 = 0 \ \ (3.6-1.96)q = 1.64q \\ z_3 = b \ \ 1.64q - 2.8q = -1.16q \end{cases} \qquad z* = \frac{F0 - RB}{q} = 1.64 \text{ m}$$

$$M_X = -F_0(z_3 + c) + R_B z_3 + q\frac{z_3^{\ 2}}{2} = \begin{cases} z_3 = 0; \ -3.6q \\ z_3 = b; \ -4.27q \end{cases} \qquad M_x^{\ *}(z^*) = -5.04q.$$

For a complex section (see point 5), the upper fibers work in tension, and the lower ones in compression. From the strength condition, taking into account that M_{Xmax}= –5.04q,

$$\begin{cases} |\sigma_{â.â.}| \le [\sigma]^+; \\ |\sigma_{í.â.}| \le [\sigma]^-, \quad where \quad [\sigma]^+ = \frac{\sigma_{Ò}^+}{n_T} = \frac{190}{1.5} = 126.7 \text{ ÌPà}, \quad [\sigma]^- = \frac{\sigma_{Ò}^-}{n_T} = \frac{210}{1.5} = 140 \text{ ÌPà}; \end{cases}$$

$$y_{â.â.} = 10 + 4.6 - 8.3 = 6.3 \text{ ñm}; \qquad y_{í.â.} = y_{\max} = 8.3 \text{ ñm};$$

а) $$|\sigma_{â.â.}| = \frac{|M_{X\max}| \cdot y_{â.â.}}{I_X} \le [\sigma]^+ \Rightarrow q^+ \le \frac{[\sigma]^+ \cdot I_X}{5.04 \cdot y_{â.â.}} = \frac{126.7 \cdot 10^6 \cdot 971 \cdot 10^{-8}}{5.04 \cdot 6.3 \cdot 10^{-2}} = 3874.6 \text{ N/m};$$

б)

$$|\sigma_{í.â.}| = \frac{|M_{X\max}| \cdot y_{í.â.}}{I_X} \le [\sigma]^- \Rightarrow q^- \le \frac{[\sigma]^- \cdot I_X}{5.04 \cdot y_{í.â.}} = \frac{140 \cdot 10^6 \cdot 971 \cdot 10^{-8}}{5.04 \cdot 8.3 \cdot 10^{-2}} = 3249.6 \ N/\text{m};$$

we take a smaller value: [q]=3249.6 N/m ≈ 3.25 kN/m, then

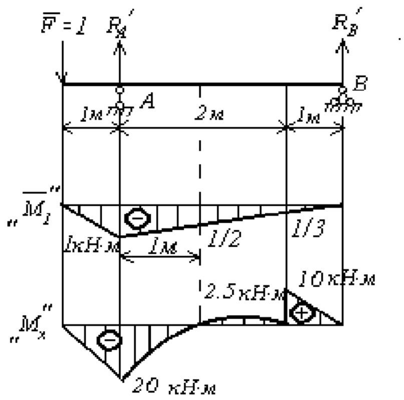
$$F_0 = 3qa = 11.70 \text{ kN};$$
$$M_0 = 2qa^2 = 9.36 \text{ kN} \cdot \text{m}.$$

It should be noted that this cross-section is located rationally, because in the upper fibers working for stretching, the absolute values of stresses are smaller than in the lower ones, while $[\sigma]^+ < [\sigma]^-$.

7. Determine the displacement of beam #4 at the point of application of force F_3. To do this, we will apply force $\overline{F} = 1$ at this point and build a plot $\overline{M}_1$.

Reactions of supports:

$$R_A{}' = \frac{4}{3}\overline{F} = \frac{4}{3};$$
$$R_B{}' = -\frac{1}{3}\overline{F} = -\frac{1}{3};$$



The plot $\overline{M}_1$ is built according to the same rules as M_X. Let's draw the plot M_X again, taking into account the value in the center of the second section of the

M_X (z_2 = 2) = 0.

We determine displacement by graphical calculation of Mohr's integral:

$$V = \frac{1}{EI_X}[M_X \times \overline{M}_1] = \frac{1}{EI_X}[\frac{1}{6}\cdot 2\cdot 1\cdot 20 + \frac{2}{6}(1\cdot 20 + \frac{1}{3}\cdot 0 + 4\cdot\frac{2}{3}\cdot 0) - \frac{1}{6}\cdot 2\cdot\frac{1}{3}\cdot 10] = \frac{12.2}{EI_X} \cdot$$

For a steel beam of I-beam section (I_X = 873 cm^4)

$$V = \frac{12.2\cdot 10^3}{2\cdot 10^{11}\cdot 873\cdot 10^{-8}} = 6{,}99\cdot 10^{-3}\text{ m} \cdot$$

The task of determining the angle of rotation of any section is solved similarly by applying a unit moment $\overline{M} = 1$.

Appendix 2 - I-beams

Number	h	b	d	t	A, cm^2	J_x, cm^4	W_x, cm^3	S_x, cm^3	J_y, cm^4	W_y, cm^3
10	100	55	4,5	7,2	12,0	198	39,7	23,0	17,9	6,49
12	120	64	4,8	7,3	14,7	350	58,4	33,7	27,9	8,72
14	140	73	4,9	7,5	17,4	572	81,7	46,8	41,9	11,5
16	160	81	5,0	7,8	20,2	873	109	62,3	58,6	14,5
18	180	90	5,1	8,1	23,4	1290	143	81,4	82,6	18,4
18a	180	100	5,1	8,3	25,4	1430	159	89,8	114	22,8
20	200	100	5,2	8,4	26,8	1840	184	104	115	23,1
20a	200	110	5,2	8,6	28,9	2030	203	114	155	28,2
22	220	110	5,4	8,7	30,6	2550	232	131	157	28,6
22a	220	120	5,4	8,9	32,8	2790	254	143	206	34,3
24	240	115	5,6	9,5	34,8	3460	289	163	198	34,5
24a	240	125	5,6	9,8	37,5	3800	317	178	260	41,6

Continuation of appendix 2

Appendix 2 - I-beams

Number	h	b	d	t	A, cm^2	J_x, cm^4	W_x, cm^3	S_x, cm^3	J_y, cm^4	W_y, cm^3
24a	240	125	5,6	9,8	37,5	3800	317	178	260	41,6
27	270	125	6,0	9,8	40,2	5010	371	210	260	41,5
27a	270	135	6,0	10,2	43,2	5500	407	229	337	50,0
30	300	135	6,5	10,2	46,5	7080	472	268	337	49,9
30a	300	145	6,5	10,7	49,9	7780	518	292	436	60,1
33	330	140	7,0	11,2	53,8	9840	597	339	419	59,9
36	360	145	7,5	12,3	61,9	1338	743	423	516	71,1
40	400	155	8,3	13,0	72,6	1906	953	545	667	86,1
45	450	160	9	14,2	84,7	2769	123	708	808	101
50	500	170	10	15,2	100	3972	158	919	104	123
55	550	180	11	16,5	118	5595	203	118	135	151
60	600	190	12	17,8	138	7680	256	149	172	182

Continuation of appendix 2

Appendix 2 – Channels

Number	h	b	d	t	A, cm^2	J_x, cm^4	W_x, cm^3	S_x, cm^3	J_y, cm^4	W_y, cm^3	Z_0 cm
5	50	32	4,4	7,0	6,16	22,8	9,1	5,59	5,61	2,75	1,16
6,5	65	36	4,4	7,2	7,51	48,6	15,0	9,0	8,7	3,68	1,24
8	80	40	4,5	7,4	8,98	89,4	22,4	13,3	12,8	4,75	1,31
10	100	46	4,5	7,6	10,9	174	34,8	20,4	20,4	6,46	1,44
12	120	52	4,8	7,8	13,3	304	50,6	29,6	31,2	8,52	1,54
14	140	58	4,9	8,1	15,6	491	70,2	40,8	45,4	11,0	1,67
14a	140	62	4,9	8,7	17,0	545	77,8	45,1	57,5	13,3	1,87
16	160	64	5,0	8,4	18,1	747	93,4	54,1	63,6	13,8	1,80
16a	160	68	5,0	9,0	19,5	823	103	59,4	78,8	16,4	2,00
18	180	70	5,1	8,7	20,7	1090	121	69,8	86	17,0	1,94
18a	184	74	5,1	9,3	22,2	1190	132	76,1	105	20,0	2,13

Continuation of appendix 2

Appendix 2 – Channels

Number	h	b	d	t	A, cm^2	J_x, cm^4	W_x, cm^3	S_x, cm^3	J_y, cm^4	W_y, cm^3	Z_0 cm
20	200	76	5,2	9,0	23,4	1520	152	87,8	113	20,	2,0
20a	200	80	5,2	9,7	25,2	1670	167	95,9	139	24,	2,2
22	220	82	5,4	9,5	26,7	2110	192	110	151	25,	2,2
22a	220	87	5,4	10,2	28,8	2330	212	121	187	30,	2,4
24	240	90	5,6	10,0	30,6	2900	242	139	208	31,	2,4
24a	240	95	5,6	10,7	32,9	3180	265	151	254	37,	2,6
27	270	95	6,0	10,5	35,2	4160	308	178	262	37,	2,4
30	300	100	6,5	11,0	40,5	5810	387	224	327	43,	2,5
33	330	105	7,0	11,7	46,5	7980	484	13,1	410	51,	2,5
36	360	110	7,5	12,6	53,4	10820	601	14,2	513	61,	2,6
40	400	115	8,0	13,5	61,5	15220	761	15,7	642	73,	2,7

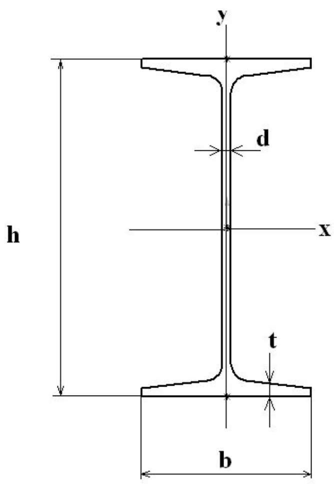


Figure 5.2 - I-beam

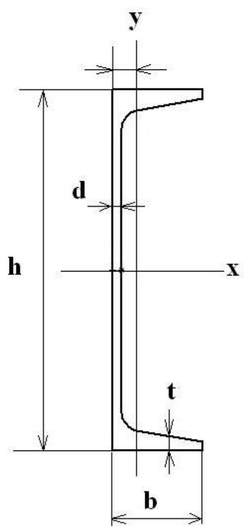


Figure 5.3 - Channel

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